PWM Inverters

Rijil Ramchand
Associate Professor
NIT Calicut

Inverters

Classifications

- Single phase & three phase
- Voltage Source & Current source
- Two-level & Multi-level

Voltage Source Inverter

Topics

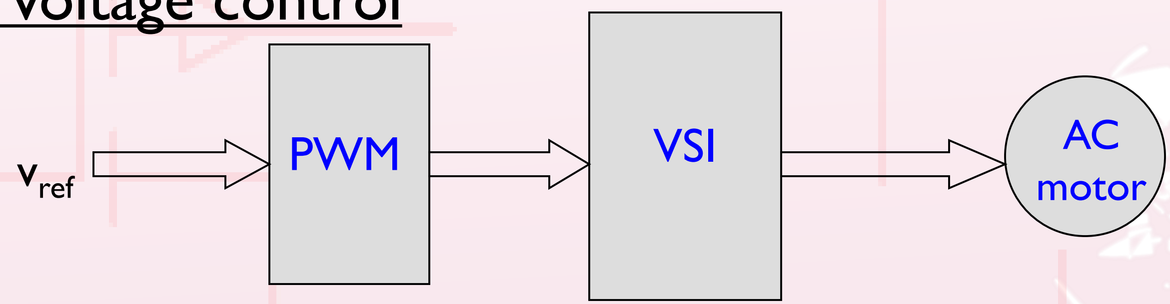
- Sinusoidal PWM
- Space vector modulation

Why Use PWM Techniques?

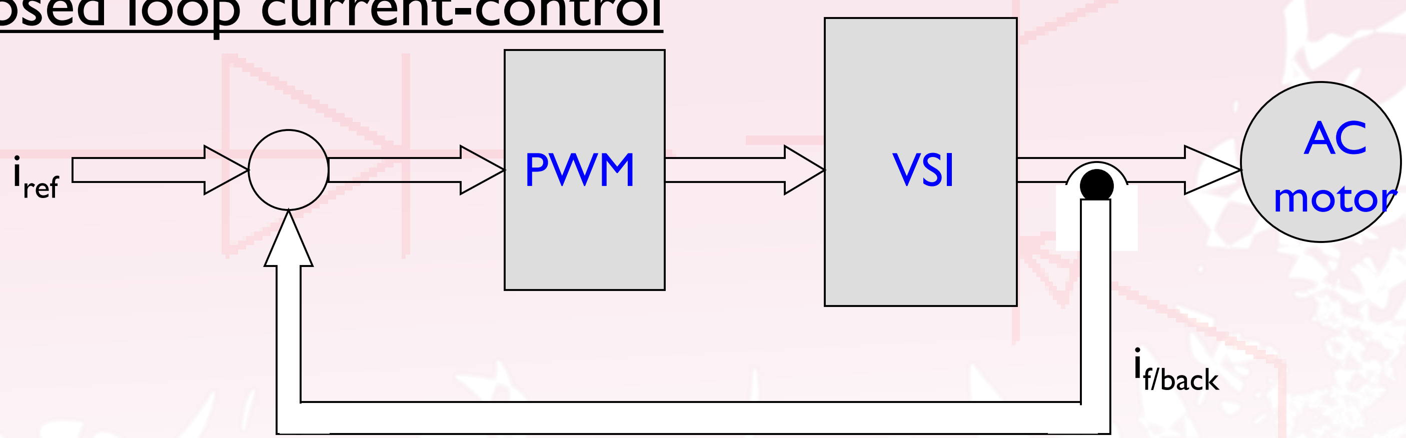
- To control inverter output frequency (fundamental)
- To control inverter output voltage (fundamental)
- To minimize harmonic distortion

Voltage Source Inverter

Open loop voltage control

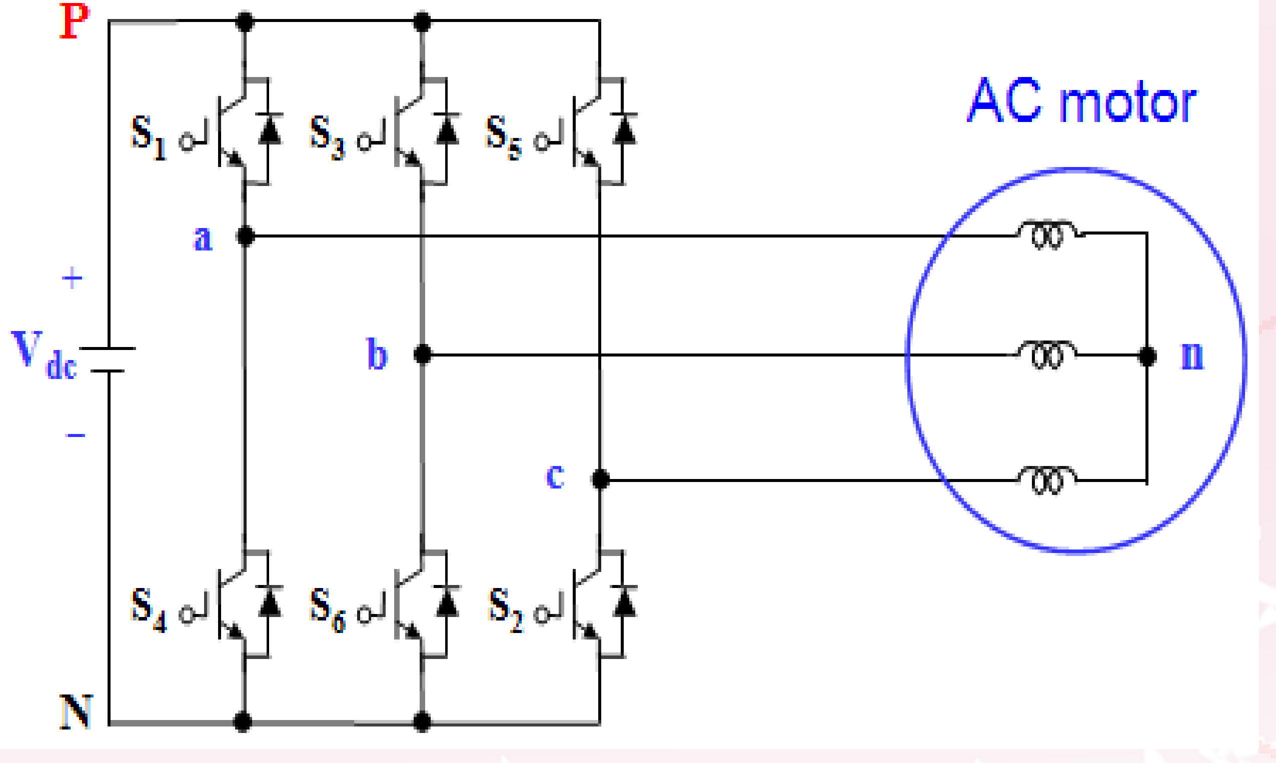


Closed loop current-control



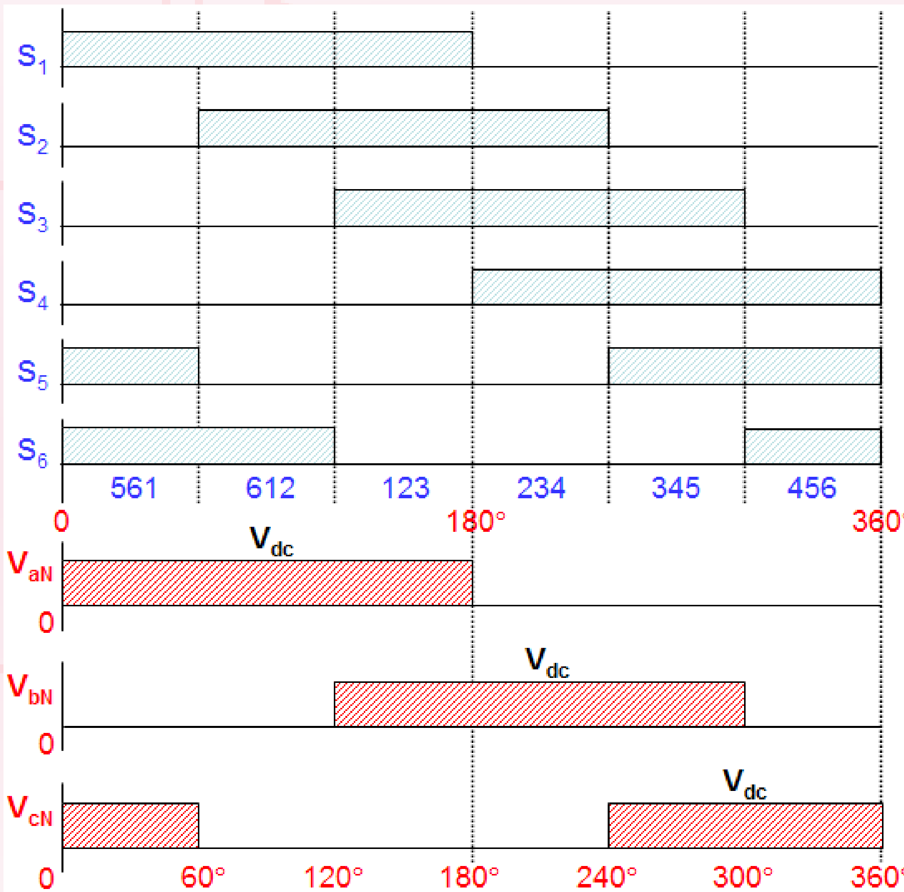
Voltage Source Inverter

Inverter Configuration



Voltage Source Inverter (VSI)
Six-Step VSI

➢ **Gating signals, switching sequence and line to negative voltages**



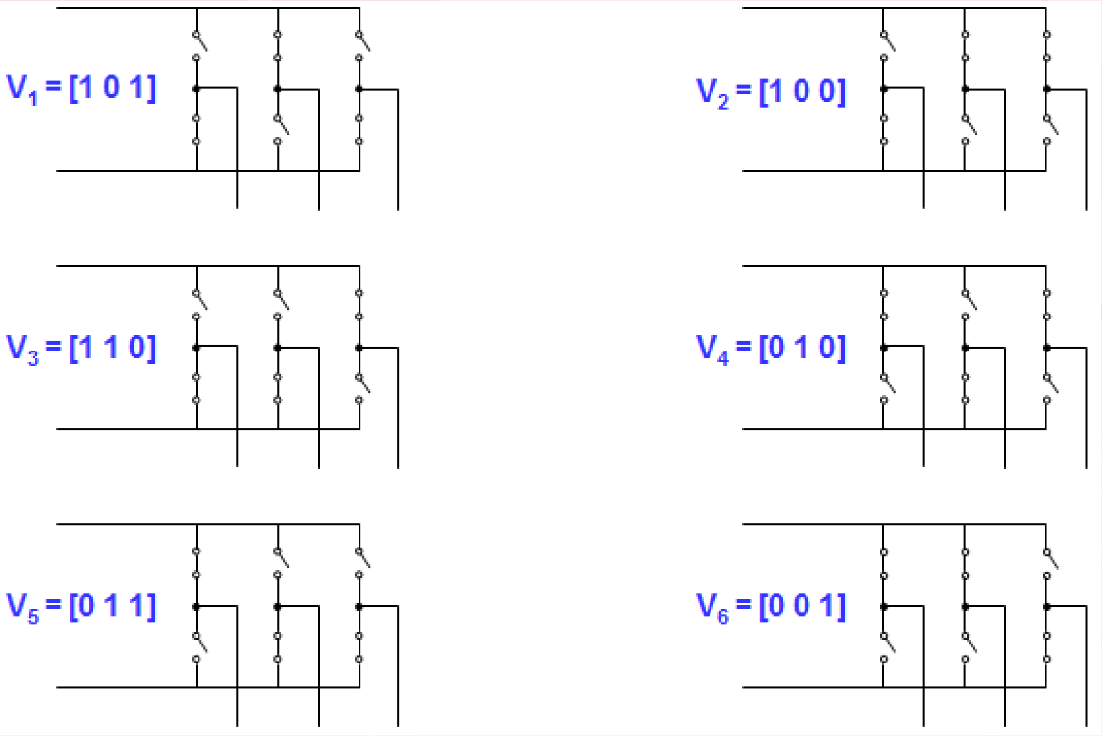
Waveforms of gating signals, switching sequence, line to negative voltages for six-step voltage source inverter.

Voltage Source Inverter (VSI)
Six-Step VSI

- **Switching Sequence:**

561 (V_1) → 612 (V_2) → 123 (V_3) → 234 (V_4) → 345 (V_5) → 456 (V_6) → 561 (V_1)

where, 561 means that S_5, S_6 and S_1 are switched on



Six inverter voltage vectors for six-step voltage source inverter.

Voltage Source Inverter (VSI)
Six-Step VSI

- Line to line voltages (V_{ab},V_{bc},V_{ca}) and line to neutral voltages (V_{an},V_{bn},V_{cn})

  - **Line to line voltages**

    ⇨ $V_{ab} = V_{aN} - V_{bN}$

    ⇨ $V_{bc} = V_{bN} - V_{cN}$

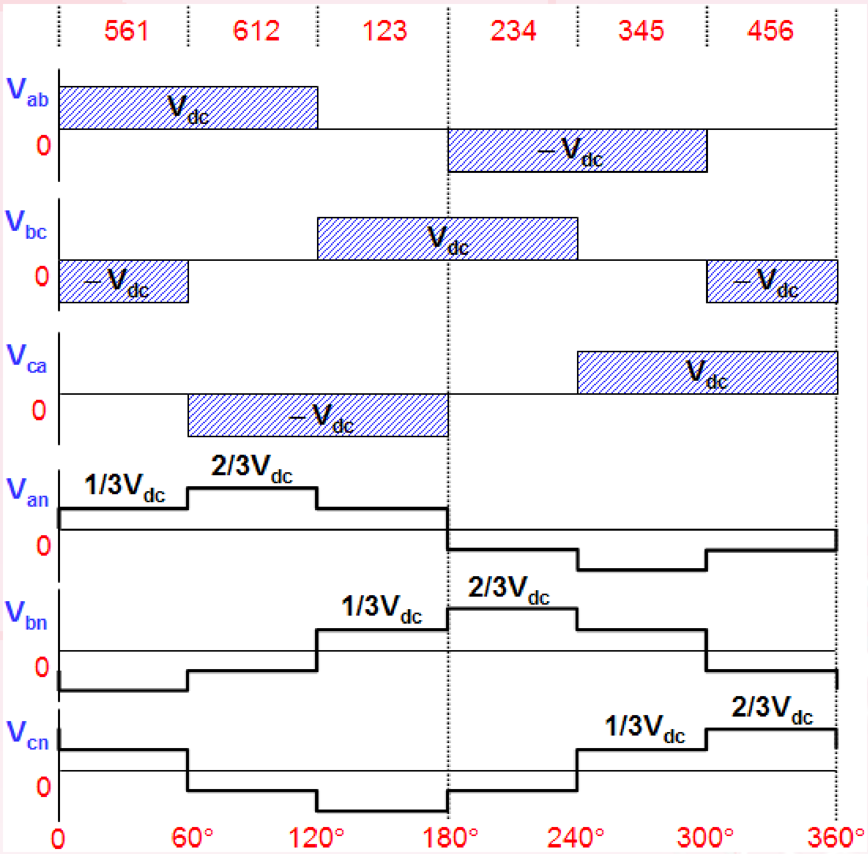
    ⇨ $V_{ca} = V_{cN} - V_{aN}$

  - **Phase voltages**

    ⇨ $V_{an} = 2/3V_{aN} - 1/3V_{bN} - 1/3V_{cN}$

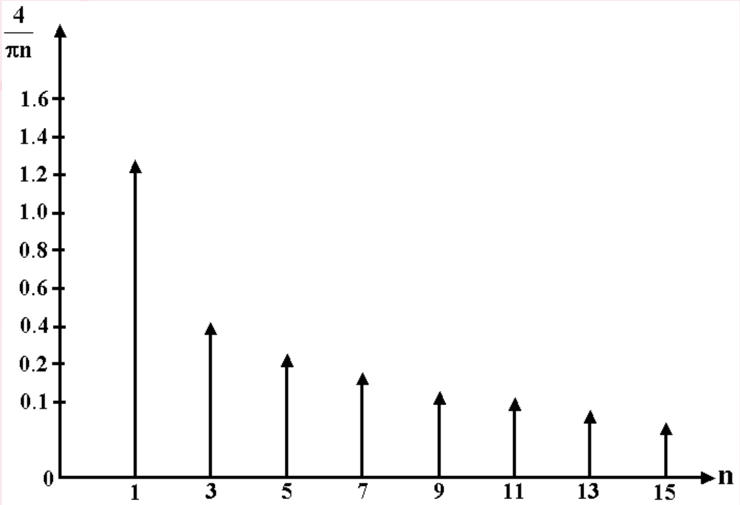
    ⇨ $V_{bn} = -1/3V_{aN} + 2/3V_{bN} - 1/3V_{cN}$

    ⇨ $V_{cn} = -1/3V_{aN} - 1/3V_{bN} + 2/3V_{cN}$



Waveforms of line to neutral (phase) voltages and line to line voltages for six-step voltage source inverter.

Voltage Source Inverter (VSI)
Six-Step VSI



Harmonic spectrum of a square wave

Voltage Source Inverter (VSI)
Six-Step VSI

- **Amplitude of line to line voltages (V_{ab}, V_{bc}, V_{ca})**
  - **Fundamental Frequency Component $(V_{ab})_1$**

$$(V_{ab})_1(\text{rms}) = \frac{\sqrt{3}}{\sqrt{2}}\frac{4}{\pi}\frac{V_{dc}}{2} = \frac{\sqrt{6}}{\pi}V_{dc} \approx 0.78V_{dc}$$

  - **Harmonic Frequency Components $(V_{ab})_h$**

  : amplitudes of harmonics decrease inversely proportional to their harmonic order

$$(V_{ab})_h(\text{rms}) = \frac{0.78}{h}V_{dc}$$

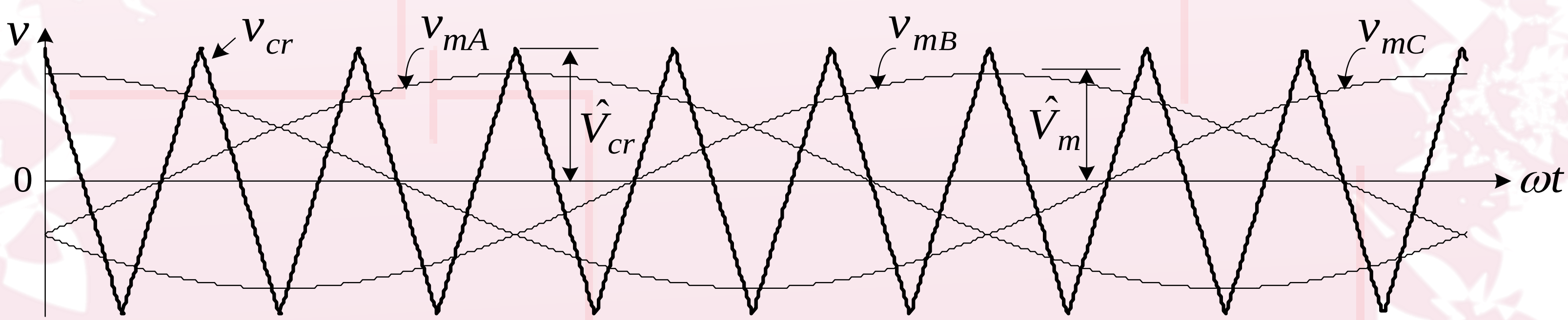
where, $h = 6n \pm 1 \quad (n = 1, 2, 3, \ldots..)$

Voltage Source Inverter (VSI)
Six-Step VSI

- **Characteristics of Six-step VSI**
  - **It is called "six-step inverter" because of the presence of six "steps" in the line to neutral (phase) voltage waveform**
  - **Harmonics of order three and multiples of three are absent from both the line to line and the line to neutral voltages and consequently absent from the currents**
  - **Output amplitude in a three-phase inverter can be controlled by only change of DC-link voltage (V_{dc})**

Sinusoidal PWM

Modulating and Carrier Waves



- v_{cr} – Carrier wave (triangle)
- v_m – Modulating wave (sine)

- Amplitude modulation index

$$m_a = \frac{\hat{V}_m}{\hat{V}_{cr}}$$

- Frequency modulation index

$$m_f = \frac{f_{cr}}{f_m}$$

Sinusoidal PWM

- m_f should be an odd integer

⇨ if m_f is not an integer, there may exist sub-hamonics at output voltage

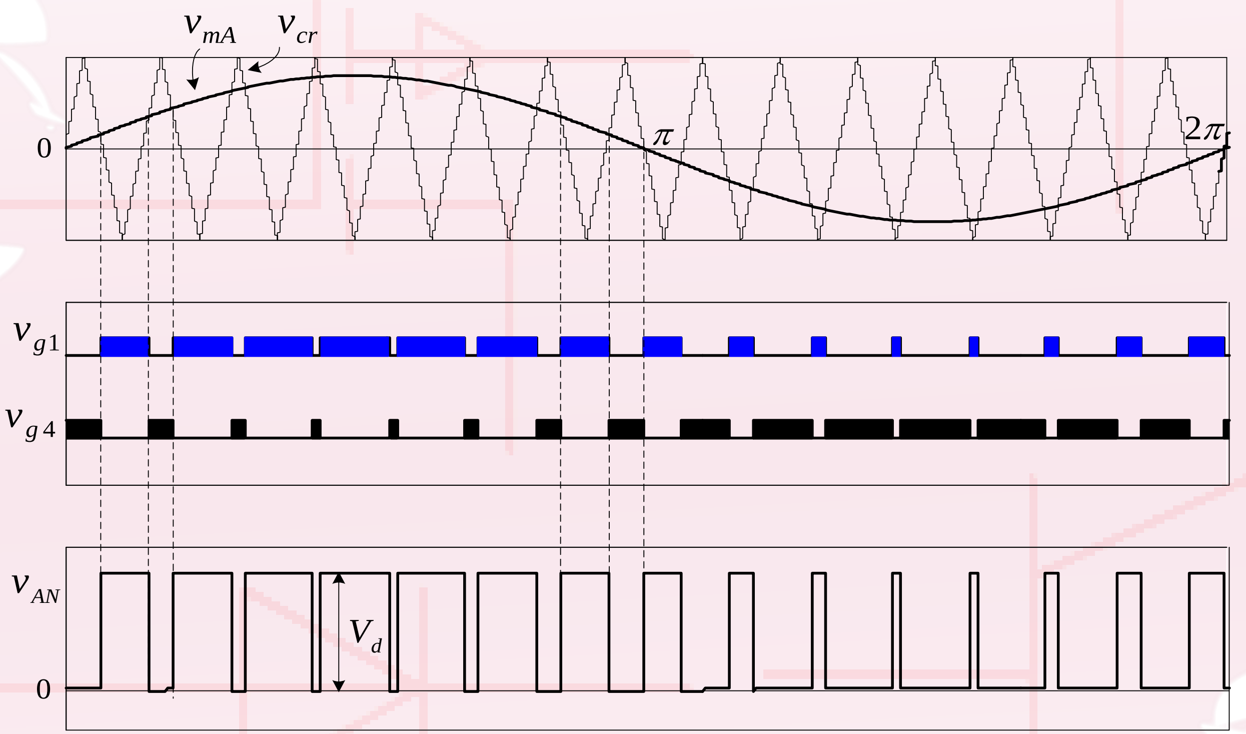
⇨ if m_f is not odd, DC component may exist and even harmonics are present at output voltage

- m_f should be a multiple of 3 for three-phase PWM inverter

⇨ An odd multiple of 3 and even harmonics are suppressed

Sinusoidal PWM

Gate Signal Generation

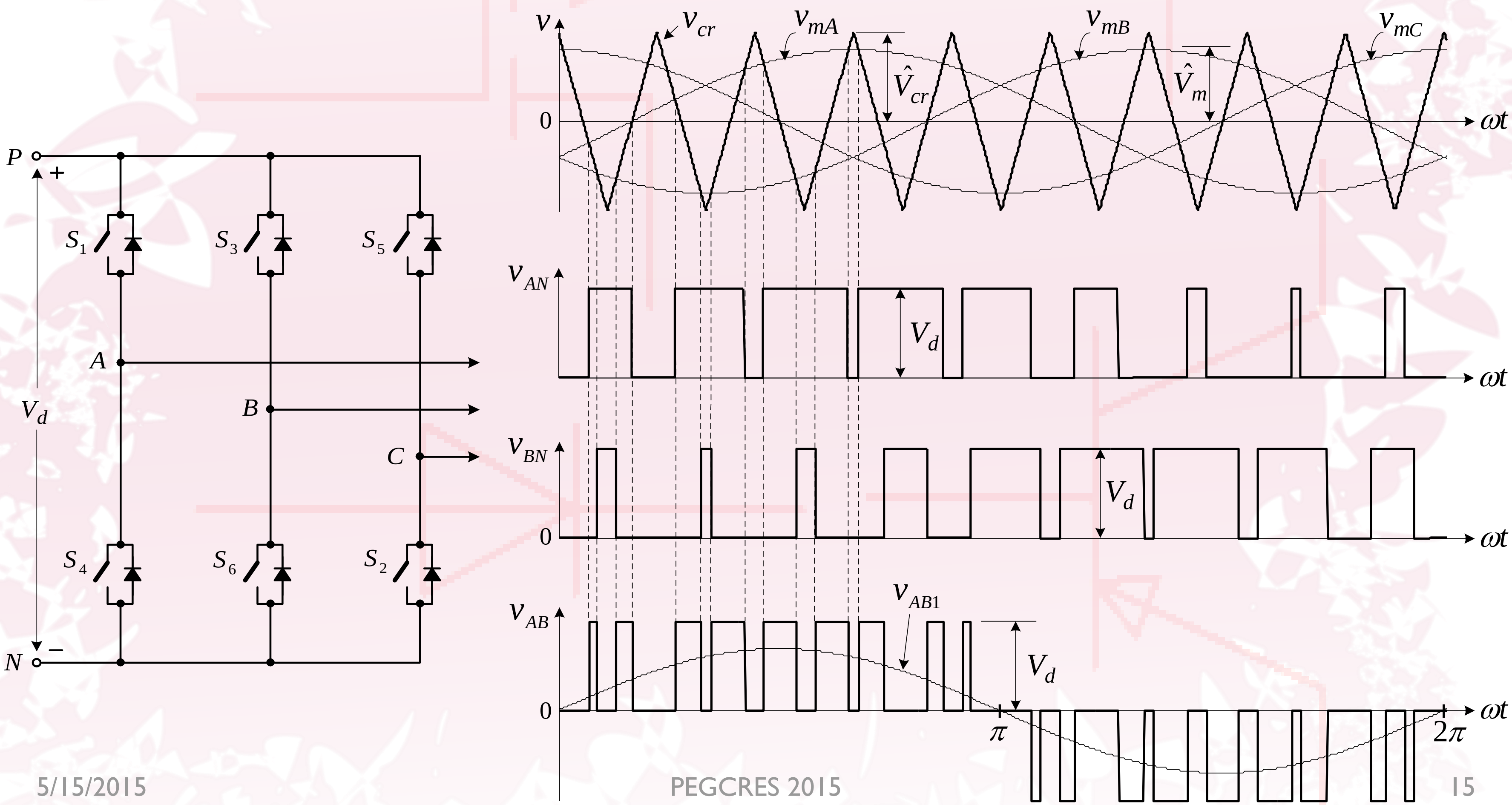


Phase A	$v_{mA} > v_{cr}$	$v_{g1} > 0 \quad (v_{g4} < 0)$	S_1 on $(S_4$ off$)$	$v_{AN} = V_d$
	$v_{mA} < v_{cr}$	$v_{g4} > 0 \quad (v_{g1} < 0)$	S_4 on $(S_1$ off$)$	$v_{AN} = 0$

V_{g1} and V_{g4} are complementary

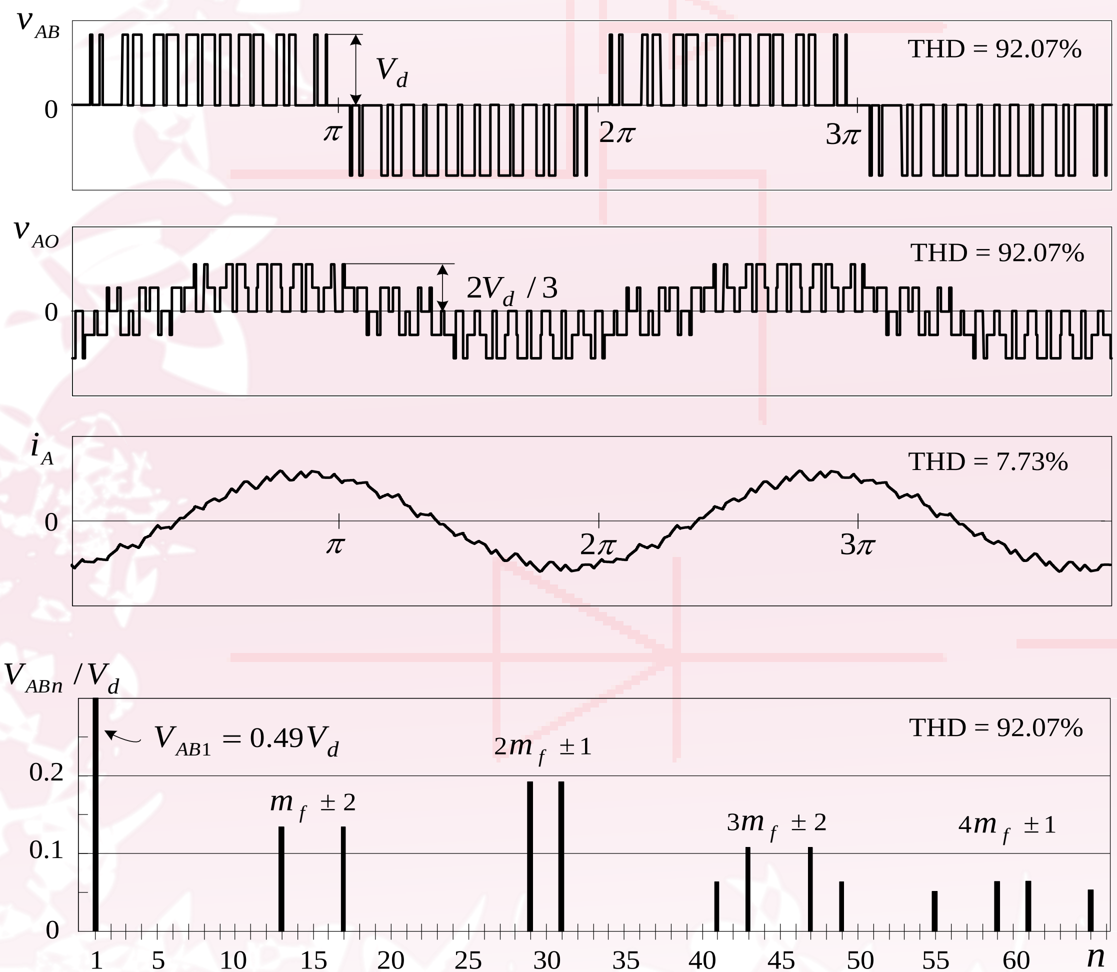
Sinusoidal PWM

Line-to-Line Voltage v_{AB}



Sinusoidal PWM

Waveforms and FFT

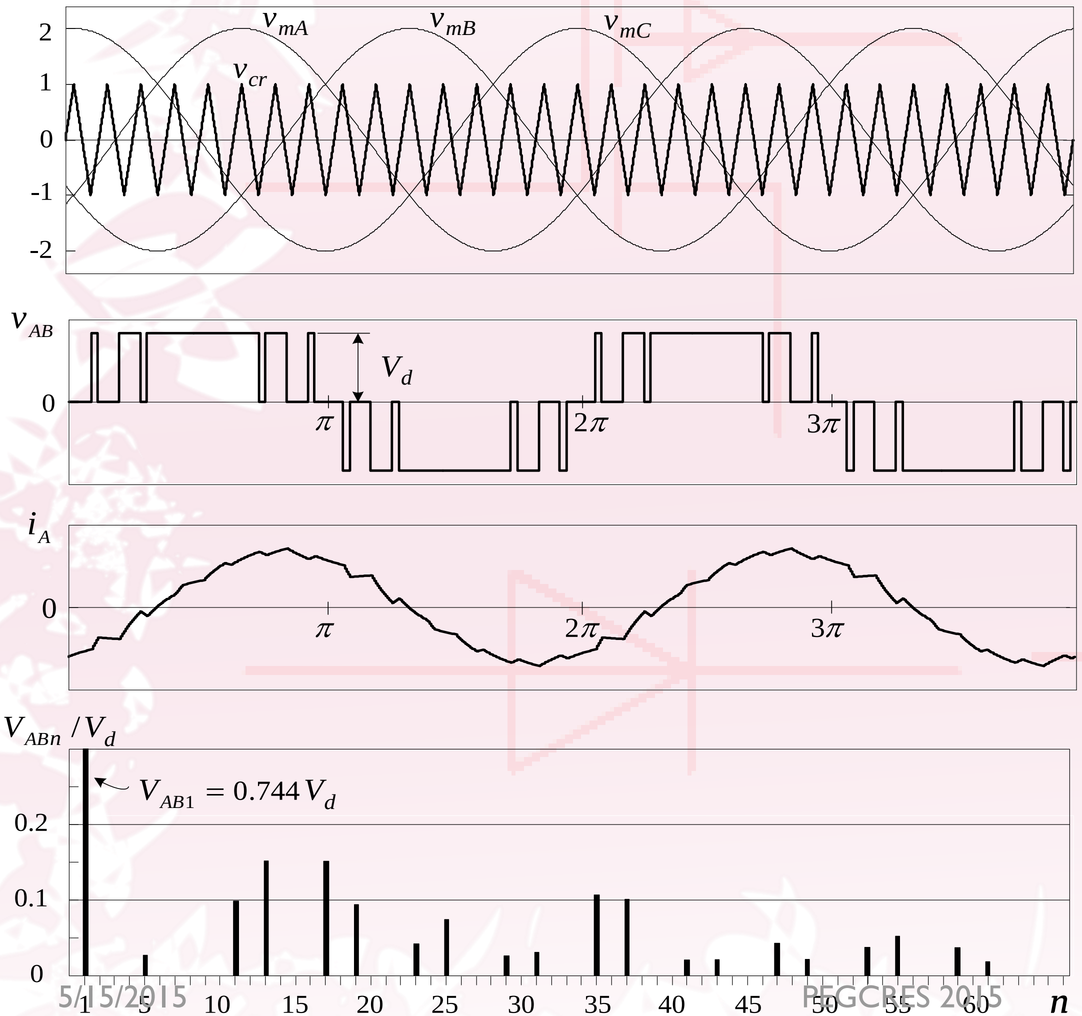


$m_a = 0.8$, $m_f = 15$,
$f_m = 60Hz$, $f_{cr} = 900Hz$

Switching frequency
$f_{sw} = f_{cr} = 900Hz$

Sinusoidal PWM

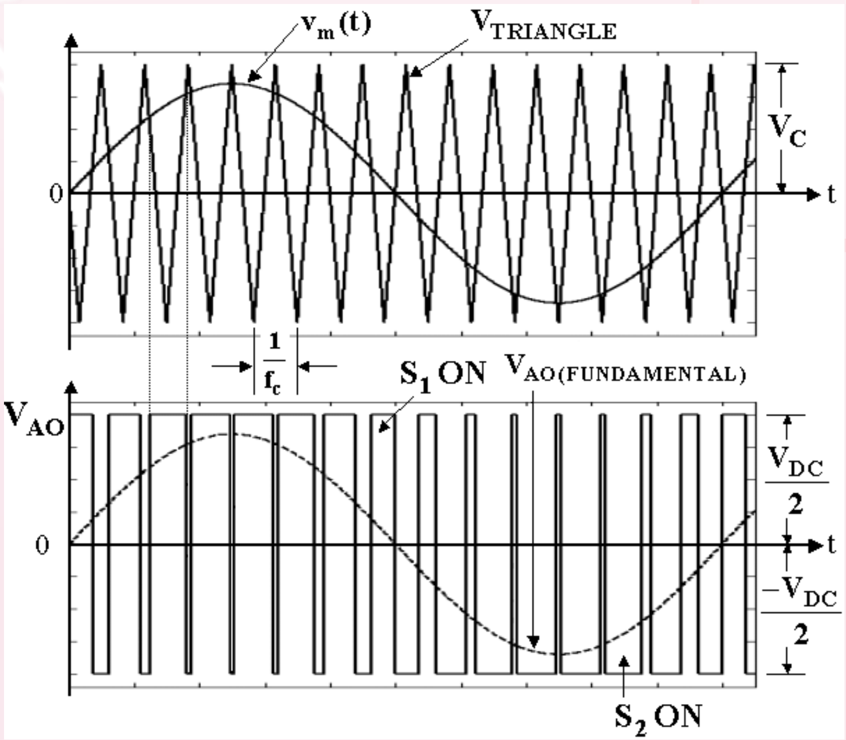
Over-Modulation



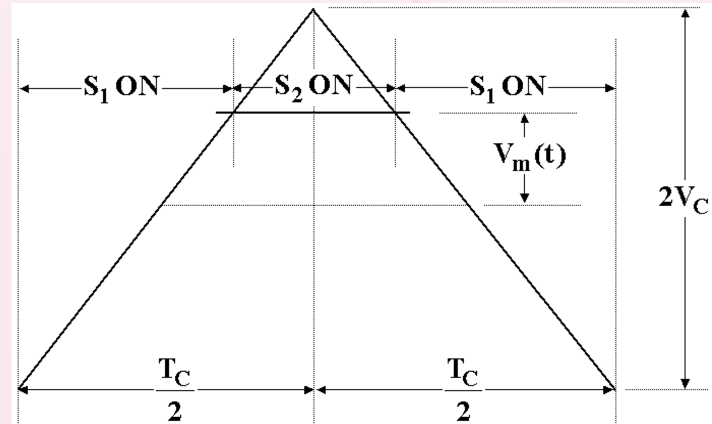
Fundamental voltage ↑

Low-order harmonics ↑

Sinusoidal PWM



(a)



$$S_1 \text{ ON period} = 2\frac{T_c}{2}\left(\frac{V_c + v_m(t)}{2Vc}\right) \rightarrow V_{AO} \text{ is } V_{DC}/2$$

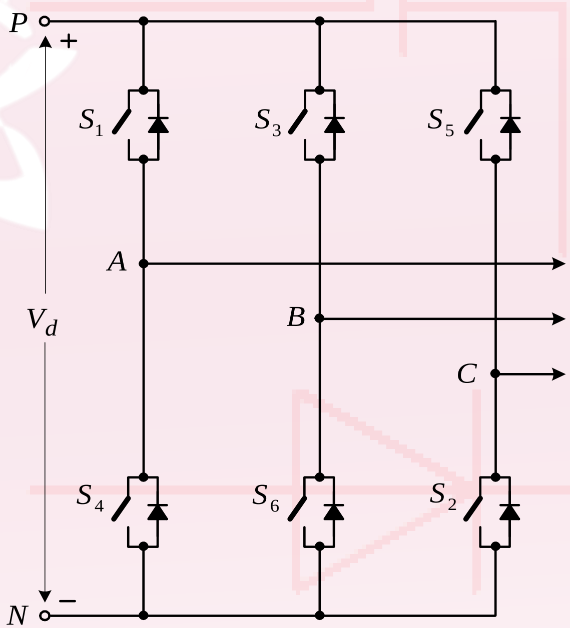
$$S_2 \text{ ON period} = T_c - T_c\left(\frac{V_c + v_m(t)}{2Vc}\right) \rightarrow V_{AO} \text{ is } -V_{DC}/2$$

$$V_{AO} \text{ average for a period } T_c = \frac{1}{T_c}\left(\frac{T_c}{2} + \frac{T_c v_m(t)}{2V_c} - \frac{T_c}{2} + \frac{T_c v_m(t)}{2V_c}\right)\frac{V_{DC}}{2}$$

$$= \frac{V_{DC}}{2}\frac{v_m(t)}{V_c}$$

Space Vector Modulation

Switching States (Three-Phase)

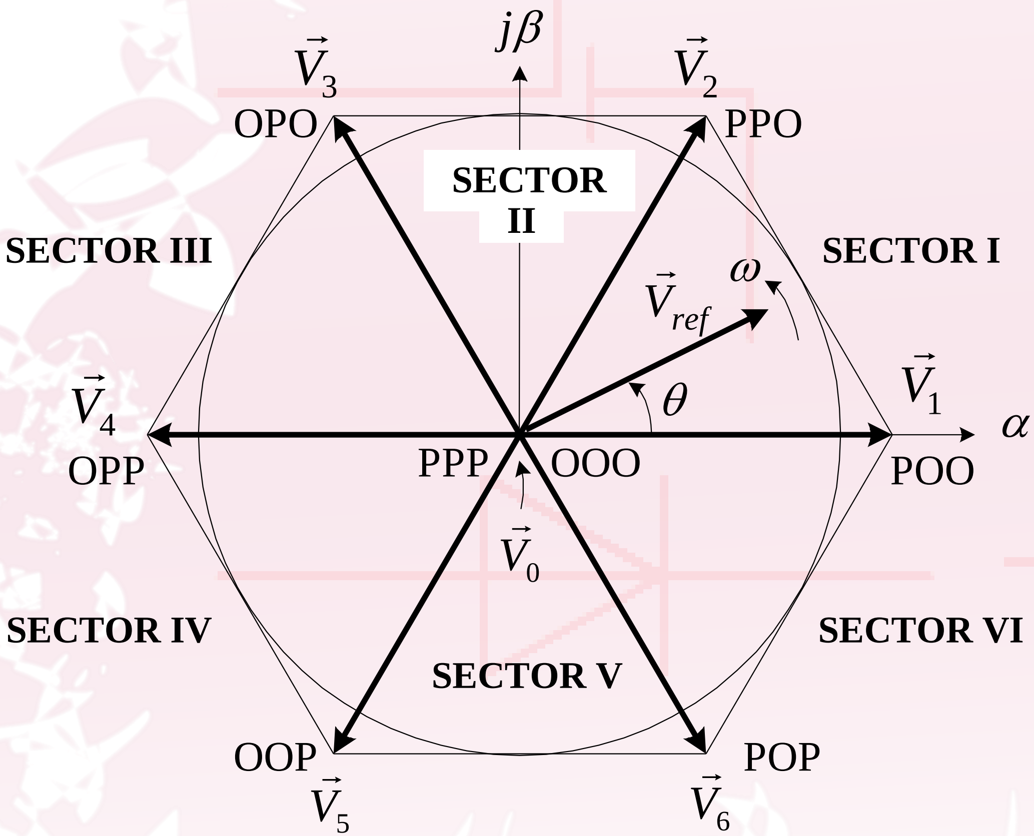


Switching State (Three Phases)	On-state Switch
[PPP]	S_1, S_3, S_5
[OOO]	S_4, S_6, S_2
[POO]	S_1, S_6, S_2
[PPO]	S_1, S_3, S_2
[OPO]	S_4, S_3, S_2
[OPP]	S_4, S_3, S_5
[OOP]	S_4, S_6, S_5
[POP]	S_1, S_6, S_5

Eight switching states

Space Vector Modulation

Space Vector Diagram



Active vectors: $\vec{V}_1$ **to** $\vec{V}_6$ **(stationary, not rotating)**

Zero vector: $\vec{V}_0$

Six sectors: ***I*** **to** ***VI***

Space Vector Modulation

Space Vectors

Three-phase voltages

$$v_{AO}(t) + v_{BO}(t) + v_{CO}(t) = 0 \qquad \textbf{(1)}$$

Two-phase voltages

$$\begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos 0 & \cos\frac{2\pi}{3} & \cos\frac{4\pi}{3} \\ \sin 0 & \sin\frac{2\pi}{3} & \sin\frac{4\pi}{3} \end{bmatrix}\begin{bmatrix} v_{AO}(t) \\ v_{BO}(t) \\ v_{CO}(t) \end{bmatrix} \qquad \textbf{(2)}$$

Space vector representation

$$\vec{V}(t) = v_\alpha(t) + j v_\beta(t) \qquad \textbf{(3)}$$

(2) → (3)

$$\vec{V}(t) = \frac{2}{3}\left[v_{AO}(t)e^{j0} + v_{BO}(t)e^{j2\pi/3} + v_{CO}(t)e^{j4\pi/3}\right] \qquad \textbf{(4)}$$

where $e^{jx} = \cos x + j\sin x$

Space Vector Modulation

Space Vectors (Example)

Switching state [POO] → S_1, S_6 and S_2 ON

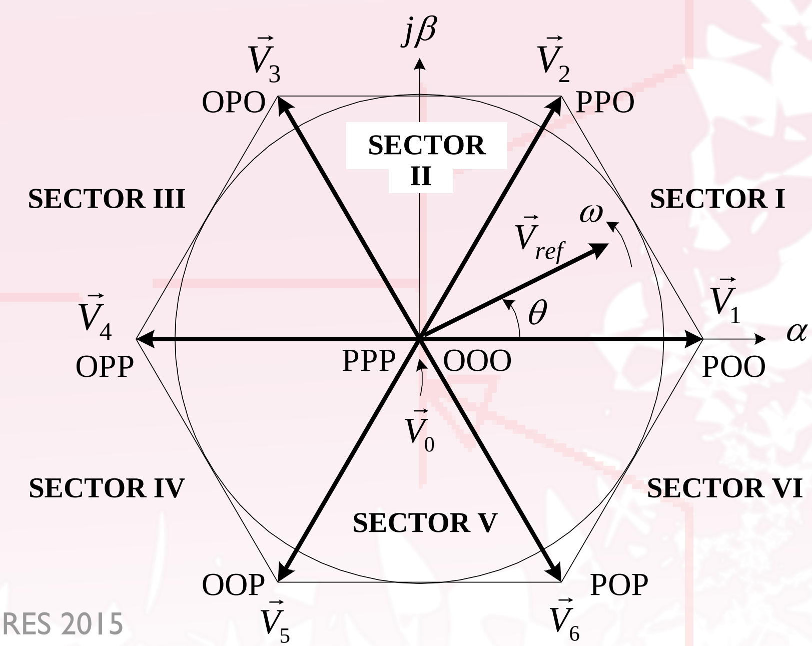
$$v_{AO}(t) = \frac{2}{3}V_d, \quad v_{BO}(t) = -\frac{1}{3}V_d, v_{CO}(t) = -\frac{1}{3}V_d$$

(5) → (4)

$$\vec{V}_1 = \frac{2}{3}V_d\, e^{j0}$$

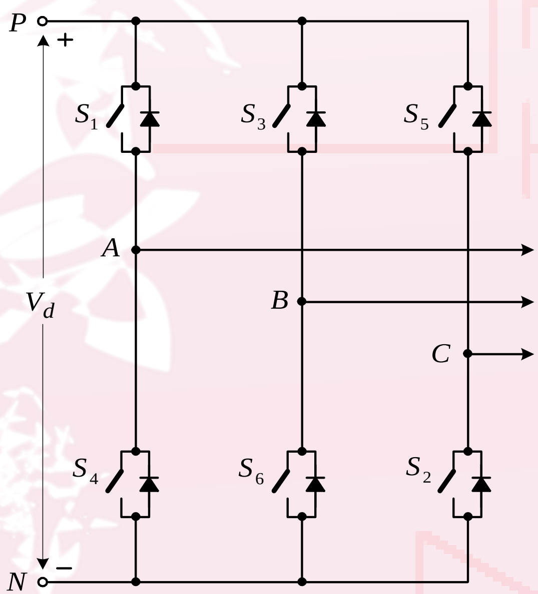
$$\vec{V}_k = \frac{2}{3}V_d\, e^{j(k-1)\frac{\pi}{3}}$$

$k = 1, 2, \ldots, 6.$



Space Vector Modulation

Active and Zero Vectors



Space Vector		Switching State (Three Phases)	On-state Switch	Vector Definition
Zero Vector	$\vec{V}_0$	[PPP]	S_1, S_3, S_5	$\vec{V}_0 = 0$
		[OOO]	S_4, S_6, S_2	
Active Vector	$\vec{V}_1$	[POO]	S_1, S_6, S_2	$\vec{V}_1 = \frac{2}{3} V_d e^{j0}$
	$\vec{V}_2$	[PPO]	S_1, S_3, S_2	$\vec{V}_2 = \frac{2}{3} V_d e^{j\frac{\pi}{3}}$
	$\vec{V}_3$	[OPO]	S_4, S_3, S_2	$\vec{V}_3 = \frac{2}{3} V_d e^{j\frac{2\pi}{3}}$
	$\vec{V}_4$	[OPP]	S_4, S_3, S_5	$\vec{V}_4 = \frac{2}{3} V_d e^{j\frac{3\pi}{3}}$
	$\vec{V}_5$	[OOP]	S_4, S_6, S_5	$\vec{V}_5 = \frac{2}{3} V_d e^{j\frac{4\pi}{3}}$
	$\vec{V}_6$	[POP]	S_1, S_6, S_5	$\vec{V}_6 = \frac{2}{3} V_d e^{j\frac{5\pi}{3}}$

Active Vector: 6

Zero Vector: 1

Redundant switching states: [PPP] and [OOO]

Space Vector Modulation

Reference Vector V_{ref}

Definition

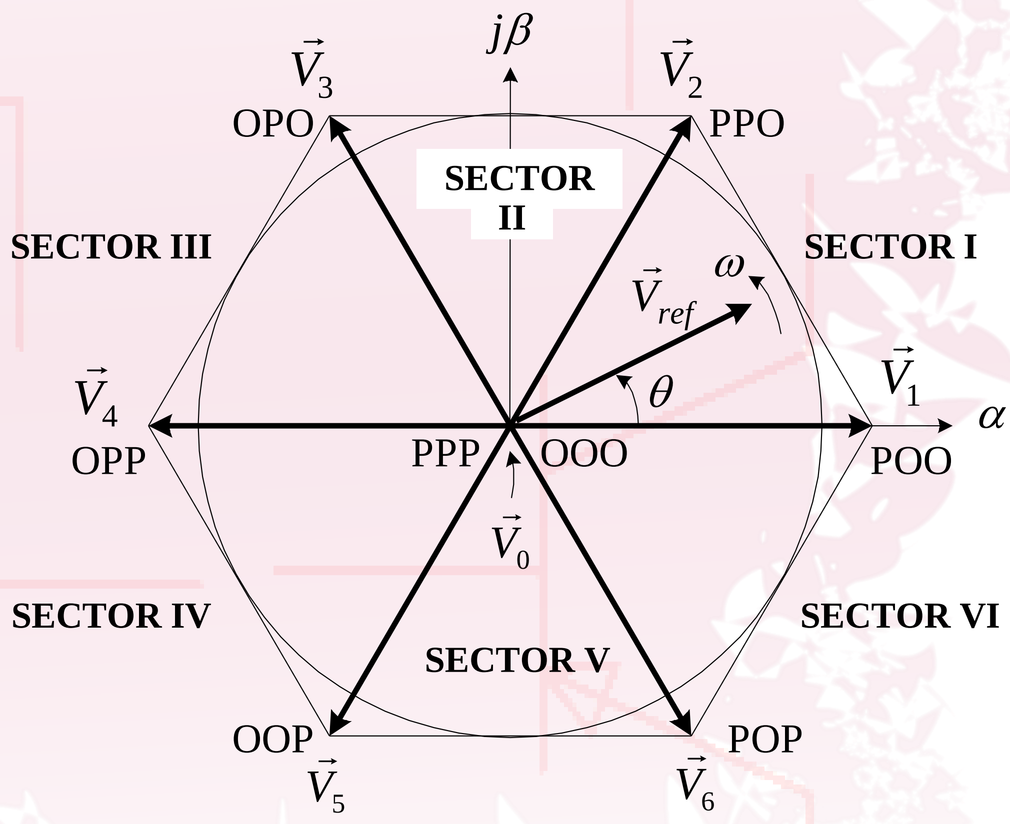
$$\vec{V}_{ref} = V_{ref}\, e^{j\theta}$$

Rotating in space at ω

$$\omega = 2\pi f \qquad \textbf{(8)}$$

Angular displacement

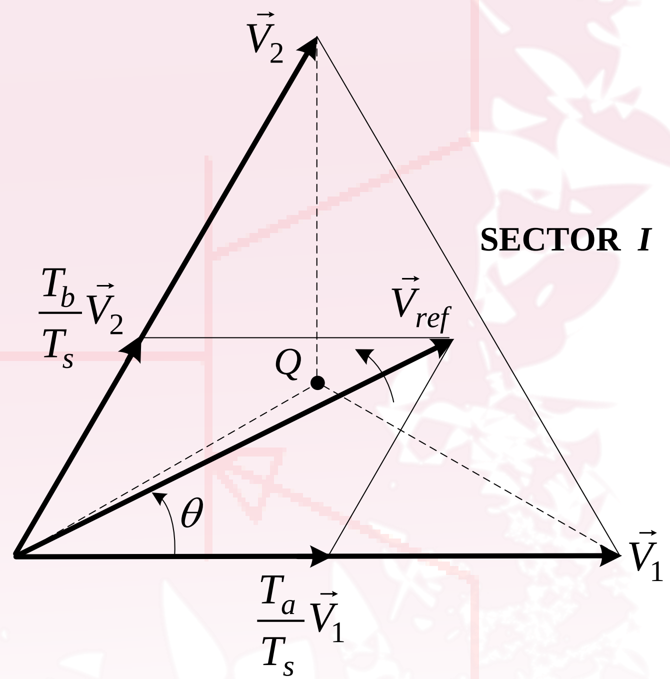
$$\theta(t) = \int_0^t \omega\, dt \qquad \textbf{(9)}$$



Space Vector Modulation

Relationship Between V_{ref} and V_{AB}

- V_{ref} is approximated by two active and zero vectors
- V_{ref} rotates one revolution, V_{AB} completes one cycle
- Length of V_{ref} corresponds to magnitude of V_{AB}

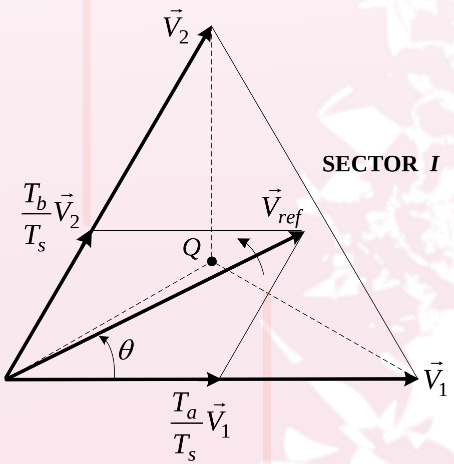


Space Vector Modulation

Dwell Time Calculation

Volt-Second Balancing

$$\begin{cases} \vec{V}_{ref} T_s = \vec{V}_1 T_a + \vec{V}_2 T_b + \vec{V}_0 T_0 \\ T_s = T_a + T_b + T_0 \end{cases} \quad \textbf{(10)}$$



***T_a, T_b* and *T_0*– dwell times for** $\vec{V}_1, \vec{V}_2$ **and** $\vec{V}_0$

***T_s*– sampling period**

Space vectors

$$\vec{V}_{ref} = V_{ref}\, e^{j\theta},\ \vec{V}_1 = \frac{2}{3} V_d\ ,\ \vec{V}_2 = \frac{2}{3} V_d\, e^{j\frac{\pi}{3}} \quad \textbf{and} \quad \vec{V}_0 = 0 \qquad \textbf{(11)}$$

(11) → (10)

$$\begin{cases} \textbf{Re:} \quad V_{ref} (\cos\theta) T_s = \frac{2}{3} V_d T_a + \frac{1}{3} V_d T_b \\ \textbf{Im:} \quad V_{ref} (\sin\theta) T_s = \frac{1}{\sqrt{3}} V_d T_b \end{cases} \qquad \textbf{(12)}$$

Space Vector Modulation

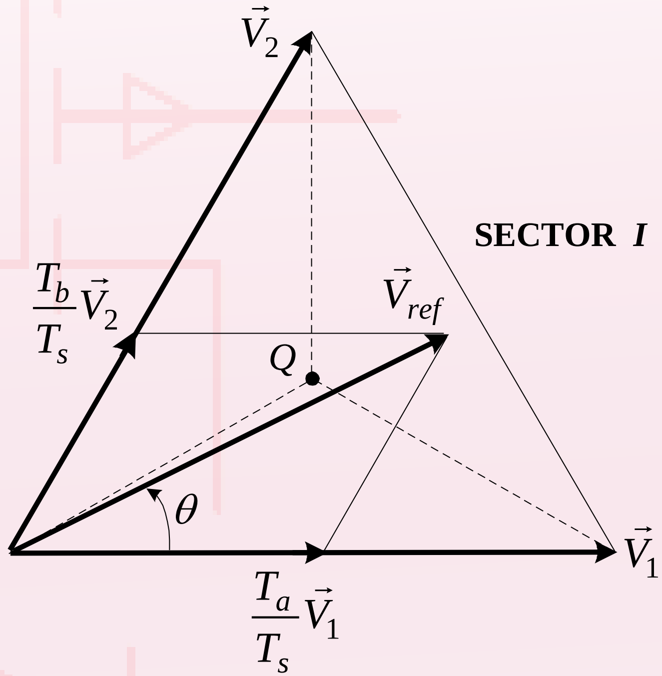
Dwell Times

Solve (12)

$$\begin{cases} T_a = \dfrac{\sqrt{3}T_s V_{ref}}{V_d}\sin(\dfrac{\pi}{3}-\theta) \\ T_b = \dfrac{\sqrt{3}T_s V_{ref}}{V_d}\sin\theta \\ T_0 = T_s - T_a - T_b \end{cases} \qquad 0 \le \theta < \pi/3 \qquad \textbf{(13)}$$

Space Vector Modulation

V_{ref} Location versus Dwell Times



$\vec{V}_{ref}$ Location	$\theta = 0$	$0 < \theta < \frac{\pi}{6}$	$\theta = \frac{\pi}{6}$	$\frac{\pi}{6} < \theta < \frac{\pi}{3}$	$\theta = \frac{\pi}{3}$
Dwell Times	$T_a > 0$ $T_b = 0$	$T_a > T_b$	$T_a = T_b$	$T_a < T_b$	$T_a = 0$ $T_b > 0$

Space Vector Modulation

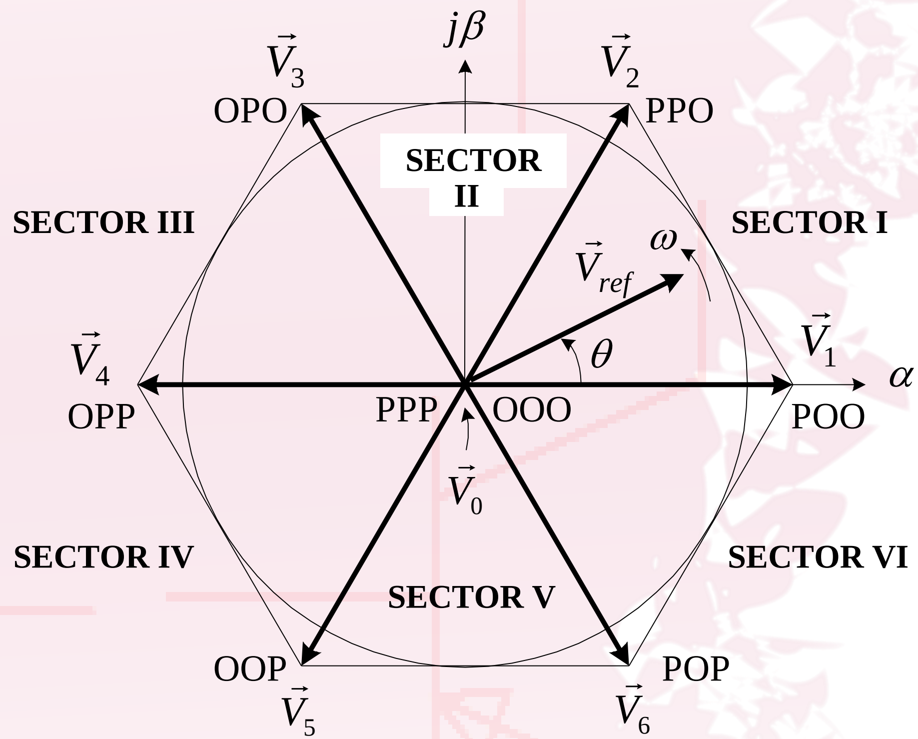
Modulation Index

$$\begin{cases} T_a = T_s m_a \sin(\frac{\pi}{3} - \theta) \\ T_b = T_s m_a \sin\theta \\ T_0 = T_s - T_b - T_c \end{cases} \quad (15)$$

$$m_a = \frac{\sqrt{3}\, V_{ref}}{V_d} \quad (16)$$

Space Vector Modulation

Modulation Range



$V_{ref,max}$

$$V_{ref,\max} = \frac{2}{3} V_d \times \frac{\sqrt{3}}{2} = \frac{V_d}{\sqrt{3}} \qquad \textbf{(17)}$$

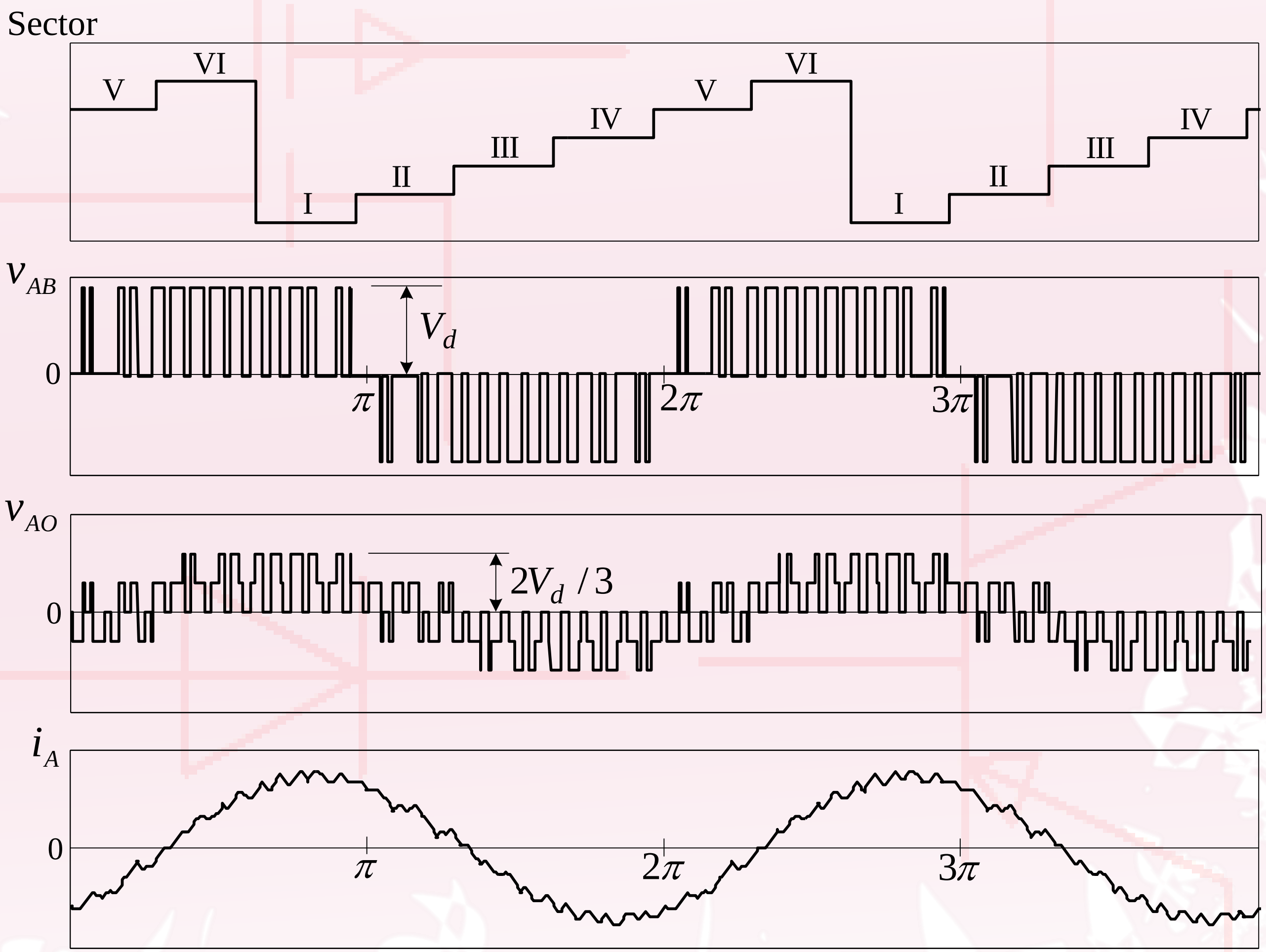
(17) → (16)

***$m_{a,max}$* = 1 →**

Modulation range: $0 \le m_a \le 1$ (18)

Space Vector Modulation

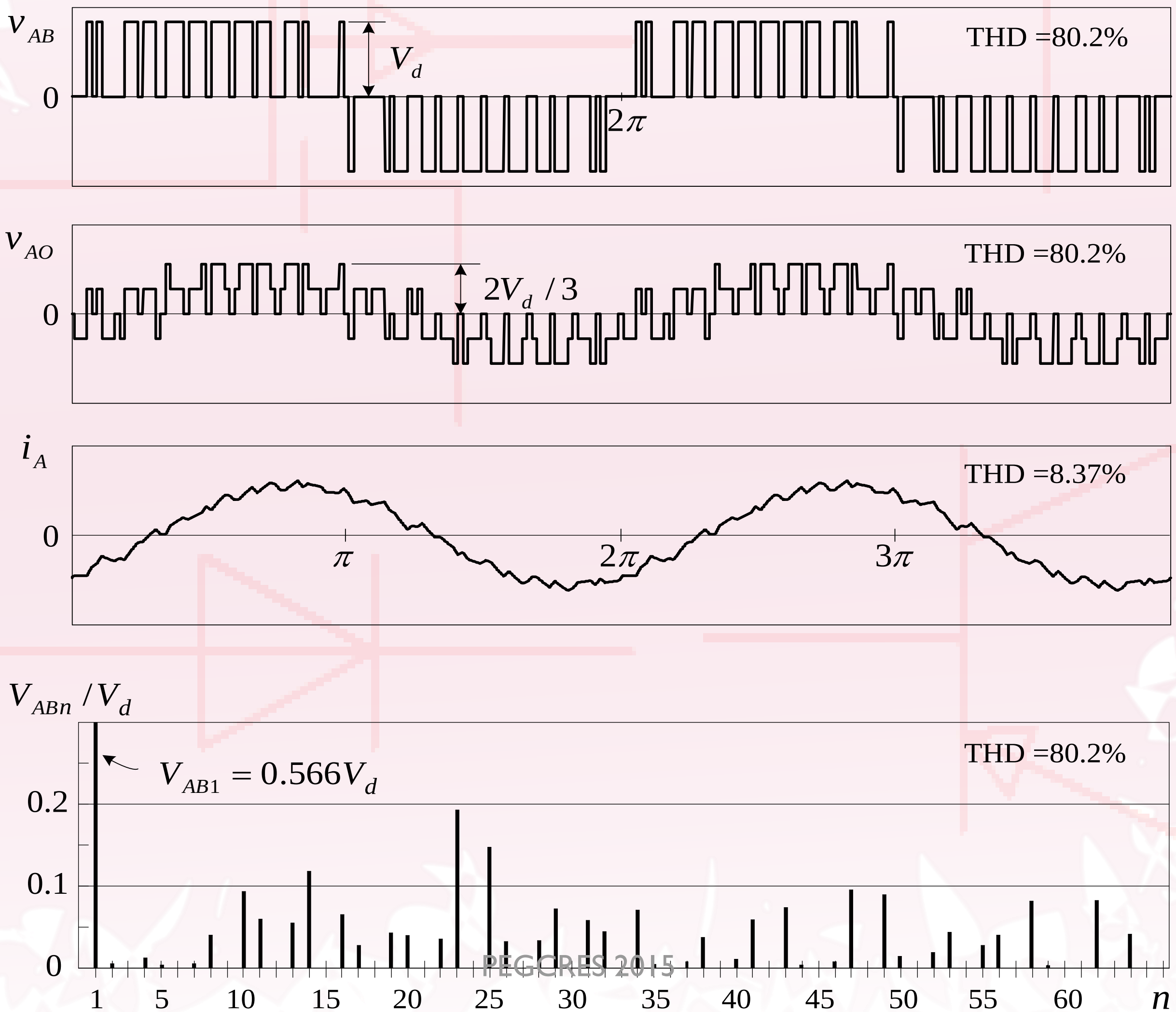
Simulated Waveforms



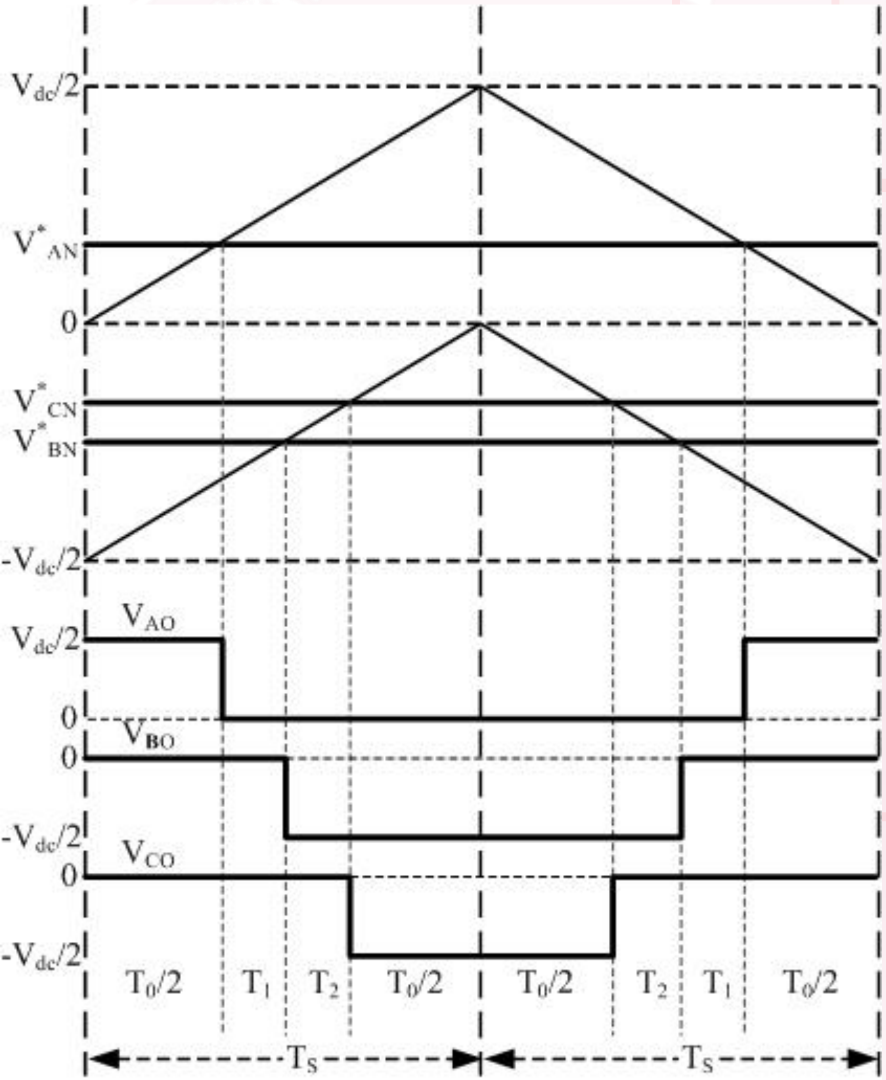
f_1 = 60Hz, f_{sw} = 900Hz, m_a = 0.696, T_s = 1.1ms

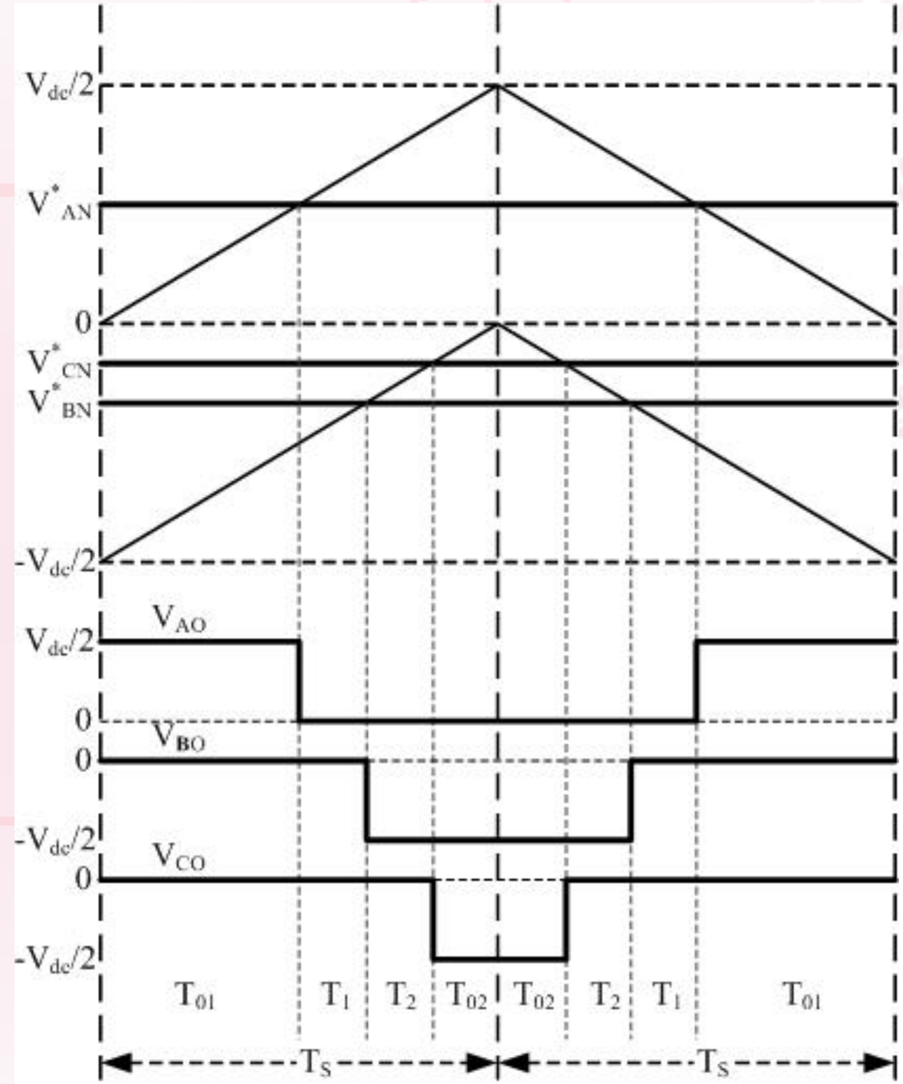
Space Vector Modulation

Waveforms and FFT



SVPWM – Modified SinePWM





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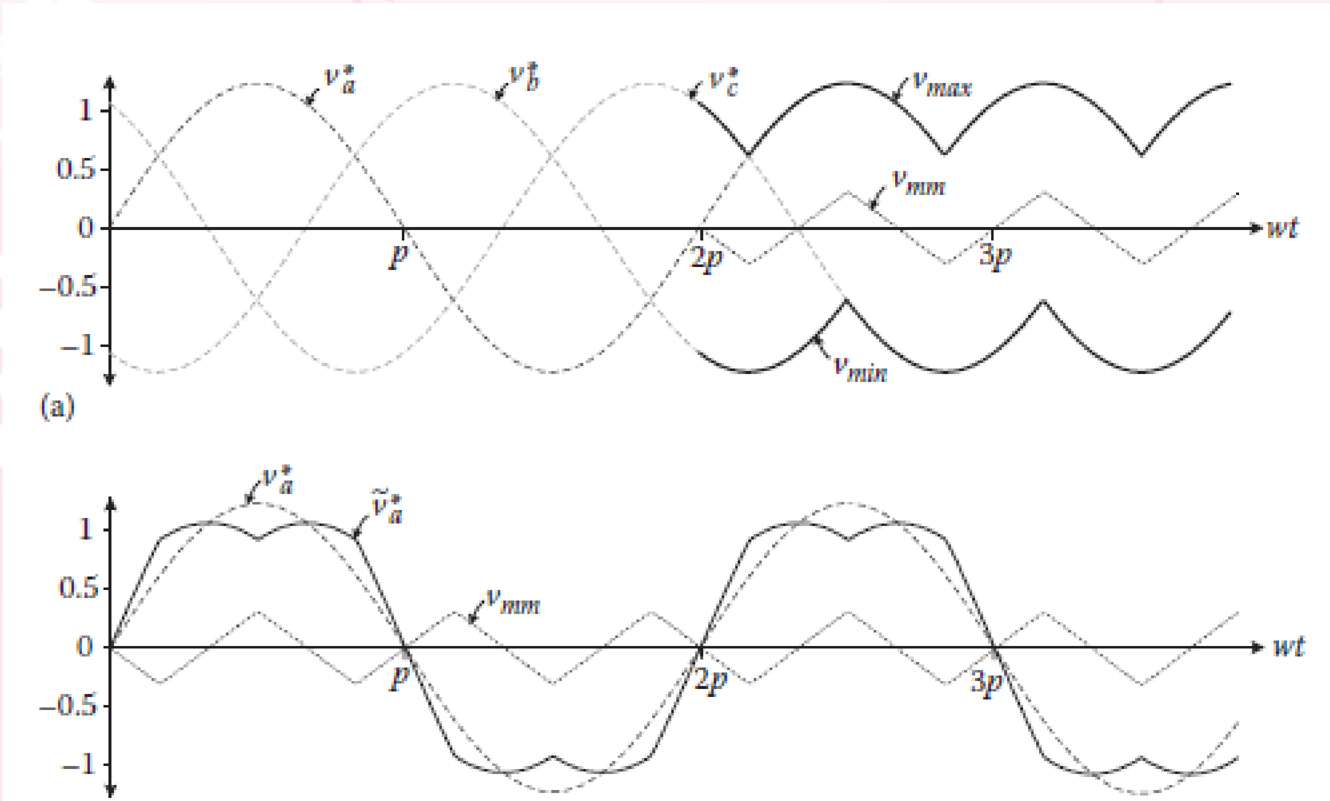

SVPWM – Modified SinePWM