# PLL and Grid Synchronization

by
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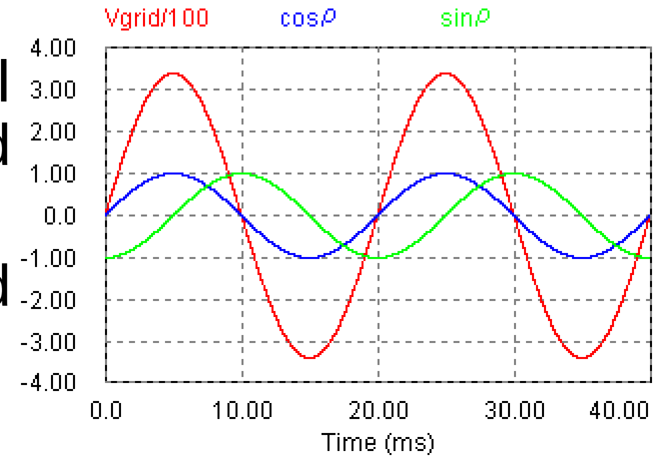
# SUMMARY OF PRESENTATION

- Unit vectors and its significance
- Basics of transformations
- Session 1: Unit Vectors For 3ϕ Balanced/Unbalanced Grid
  - ✓ Selection of reference variable for control
  - ✓ Selection of output parameter for PI-controller
  - ✓ Design of PI-controller constants
  - ✓ System bandwidth and method to select bandwidth
  - ✓ Unit vector under unbalanced grid condition
  - ✓ Test results
  - ✓ A Simple method for unit vector construction for balanced grid
- Session 2: Unit Vectors For 1ϕ Grid
  - ✓ Constructing two unit magnitude $90^0$ displaced components
  - ✓ Mitigating the effect of grid frequency variation
    - ✓ Approximation method
    - ✓ Rigorous method
  - ✓ Test results

# Unit Vectors

**What is unit vector ?**

- Two unity magnitude fundamental sinusoidal quantities, which are displaced by $90^0$ from each other
- One of the unit vector is in phase with grid voltage
- This should be free from harmonics
- Phase angle error from grid voltage should be minimum




**Significance of unit vector ?**

- STATCOM is an independent voltage source
- Unit vector helps to synchronize STATCOM voltage and Grid voltage
- Also unit vector is used to extract the active and reactive power component separately
- It is also used to separate the individual harmonics in case of active power filters

# Transformations

- 3ϕ to α-β transformations

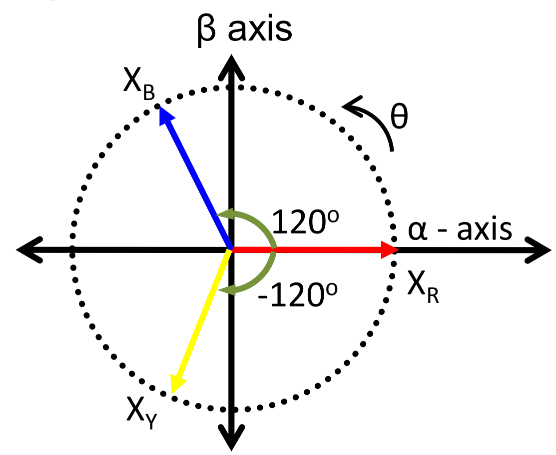
$$X_\alpha = X_R - \frac{1}{2}(X_Y + X_B)$$

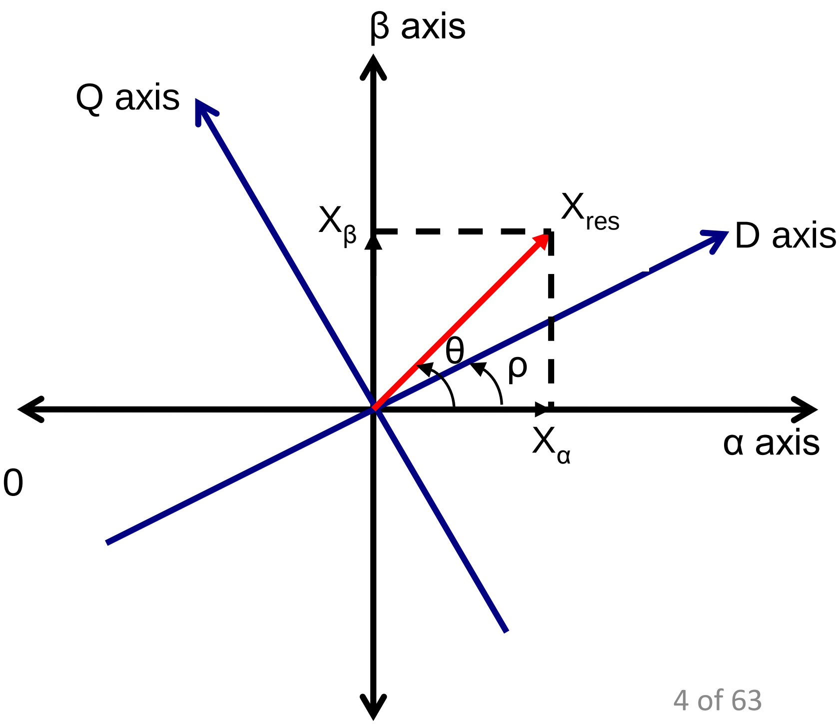
$$X_\beta = \frac{\sqrt{3}}{2}(X_Y - X_B)$$

- α-β to D-Q transformations

$$\begin{bmatrix} X_d \\ X_q \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ -U_2 & U_1 \end{bmatrix} \begin{bmatrix} X_\alpha \\ X_\beta \end{bmatrix}$$

- $U_1$ and $U_2$ unity magnitude components, where $U_2$ lag $U_1$ by $90^0$
- For grid connected system $U_1$ is aligned along R-phase grid voltage



# SESSION 1

## UNIT VECTORS FOR THREE PHASE BALANCED/UNBALANCED GRID

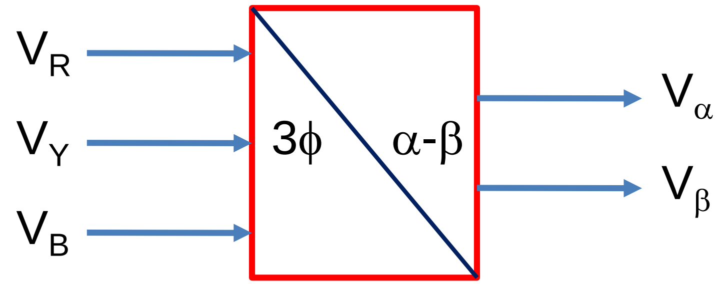
# Unit vector for Balanced/Unbalanced Grid condition

- Let the grid voltages are,

$$V_R = V_g \sin \omega_g t$$

$$V_Y = V_g \sin(\omega_g t - 120)$$

$$V_B = V_g \sin(\omega_g t + 120)$$




- Objective,

$$U_1 = \sin \omega_g t$$

$$U_2 = -\cos \omega_g t$$

- Apply three phase to two phase (α-β) transformation,

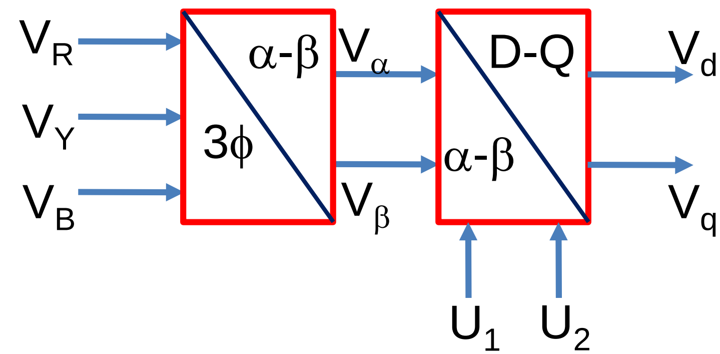
$$V_\alpha = V_R - \frac{1}{2}(V_Y + V_B) = \frac{3}{2} V_g \sin \omega_g t$$

$$V_\beta = \frac{\sqrt{3}}{2}(V_Y - V_B) = -\frac{3}{2} V_g \cos \omega_g t$$

# Unit vector for Balanced/Unbalanced Grid condition

- If $U_1$ and $U_2$ are known then $V\alpha$ and $V_\beta$ can be transformed to D-Q axis
- Assume $U_1$ and $U_2$ are known, then

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} U_1 & U_2 \\ -U_2 & U_1 \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$$




- Let $U_1$ is not synchronized to $V_R$ and its frequency is $\omega_g'$, then,

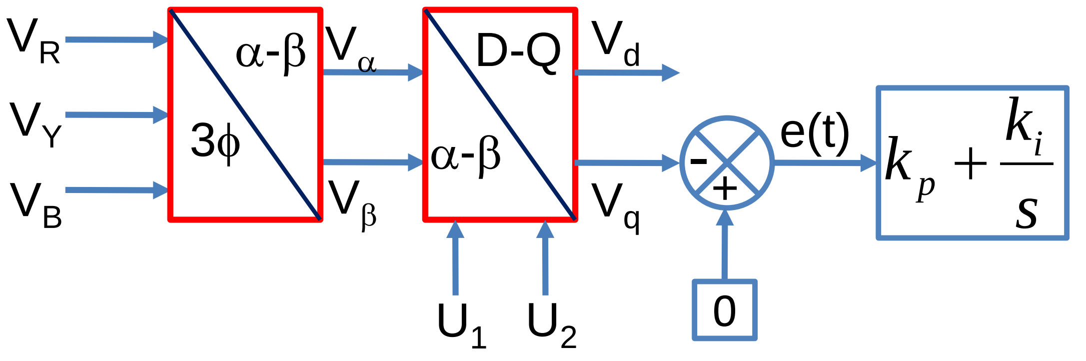
$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin \omega_g' t & -\cos \omega_g' t \\ \cos \omega_g' t & \sin \omega_g' t \end{bmatrix} \begin{bmatrix} \frac{3}{2} V_g \sin \omega_g t \\ -\frac{3}{2} V_g \cos \omega_g t \end{bmatrix} = \begin{bmatrix} \frac{3}{2} V_g \cos((\omega_g - \omega_g')t) \\ \frac{3}{2} V_g \sin((\omega_g - \omega_g')t) \end{bmatrix}$$

- When $U_1$ gets synchronized with $V_R$, then $\omega_g = \omega_g'$ and,

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \frac{3}{2} V_g \\ 0 \end{bmatrix}$$

# Unit vector for Balanced/Unbalanced Grid condition

- If $V_q = 0$ is ensured, then $U_1$ will be synchronized to $V_R$
- To ensure $V_q = 0$, feedback with simple PI-controller can be used

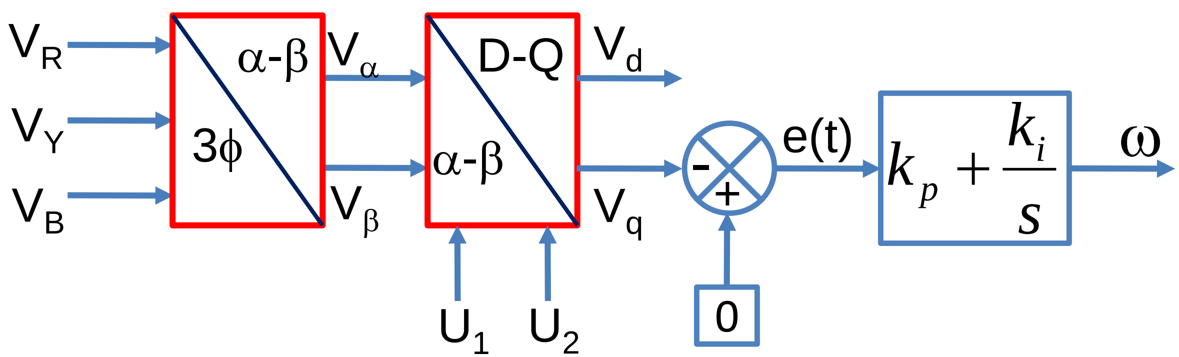



- PI-controller will be sufficient,
  - Since $V_d$ and $V_q$ are d.c. quantities and variation in e(t) with time is minimal

# Unit vector for Balanced/Unbalanced Grid condition

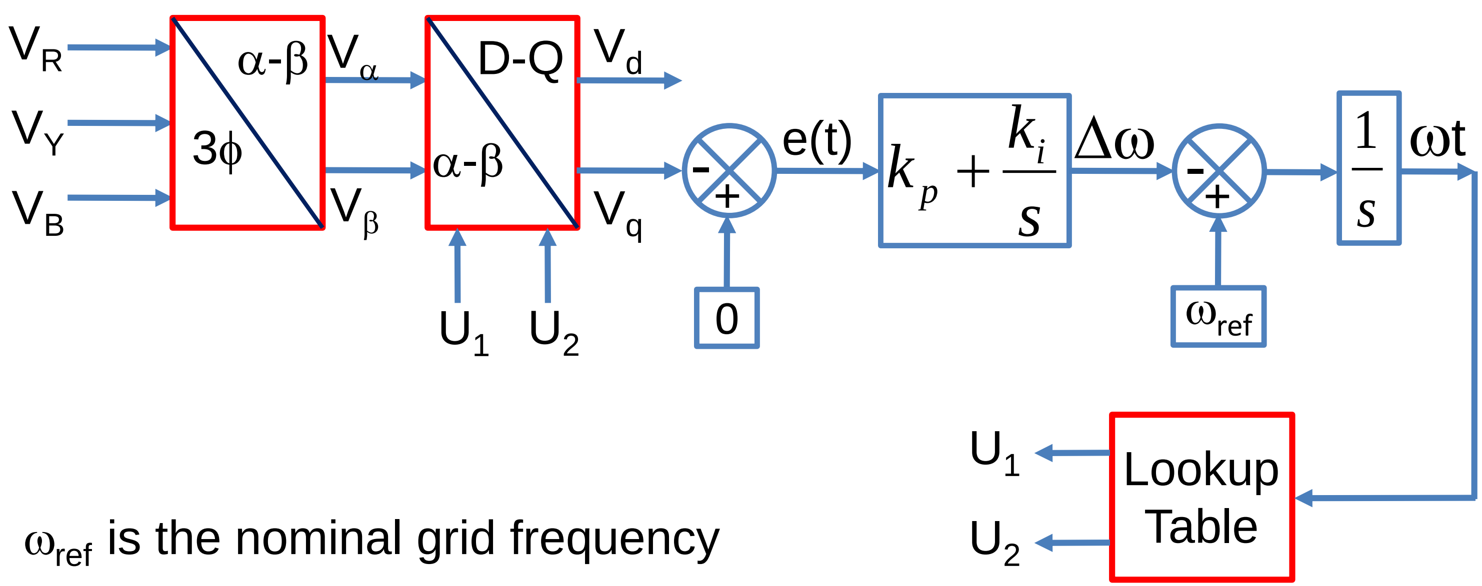
- What should be the output parameter of PI-controller ?
  - PI-controller performs well when the selected output parameter magnitude swing is minimal
  - Our objective is to obtain $U_1$ and $U_2$, but they are sinusoidally varying quantities and results in large swing in magnitude
  - The angle of $U_1$ and $U_2$ are also will be varying with time in large extend
  - Instead the frequency of $U_1$ and $U_2$ can be thought of selecting as output parameter of PI-controller

$$U_1 = \sin \omega t, \quad U_2 = -\cos \omega t$$



# Unit vector for Balanced/Unbalanced Grid condition

- What should be the output parameter of PI-controller ?
  - If the grid frequency variation is minimal, then
  - PI-controller performance can be further improved by selecting the output of PI-controller as $\Delta\omega$ (variation in grid frequency) instead of absolute grid frequency, $\omega$




$\omega_{ref}$ is the nominal grid frequency

# Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants ?
  - At time t=0, let grid and unit vectors ($U_1$ and $U_2$) frequency be $\omega_g$
  - At time t=$0^+$, grid frequency changed from $\omega_g$ to $\omega'_g$ and unit vector frequency remains at $\omega_g$
  - $3\phi$ to $\alpha$-$\beta$ transformation of grid voltage is,

$$V_\alpha = \frac{3}{2} V_g \sin \omega'_g t, \quad V_\beta = -\frac{3}{2} V_g \cos \omega'_g t$$

  - $\alpha$-$\beta$ to D-Q transformation gives,

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin \omega_g t & -\cos \omega_g t \\ \cos \omega_g t & \sin \omega_g t \end{bmatrix} \begin{bmatrix} \frac{3}{2} V_g \sin \omega'_g t \\ -\frac{3}{2} V_g \cos \omega'_g t \end{bmatrix} = \begin{bmatrix} \frac{3}{2} V_g \cos((\omega'_g - \omega_g) t) \\ \frac{3}{2} V_g \sin((\omega'_g - \omega_g) t) \end{bmatrix}$$

# Unit vector for Balanced/Unbalanced Grid condition

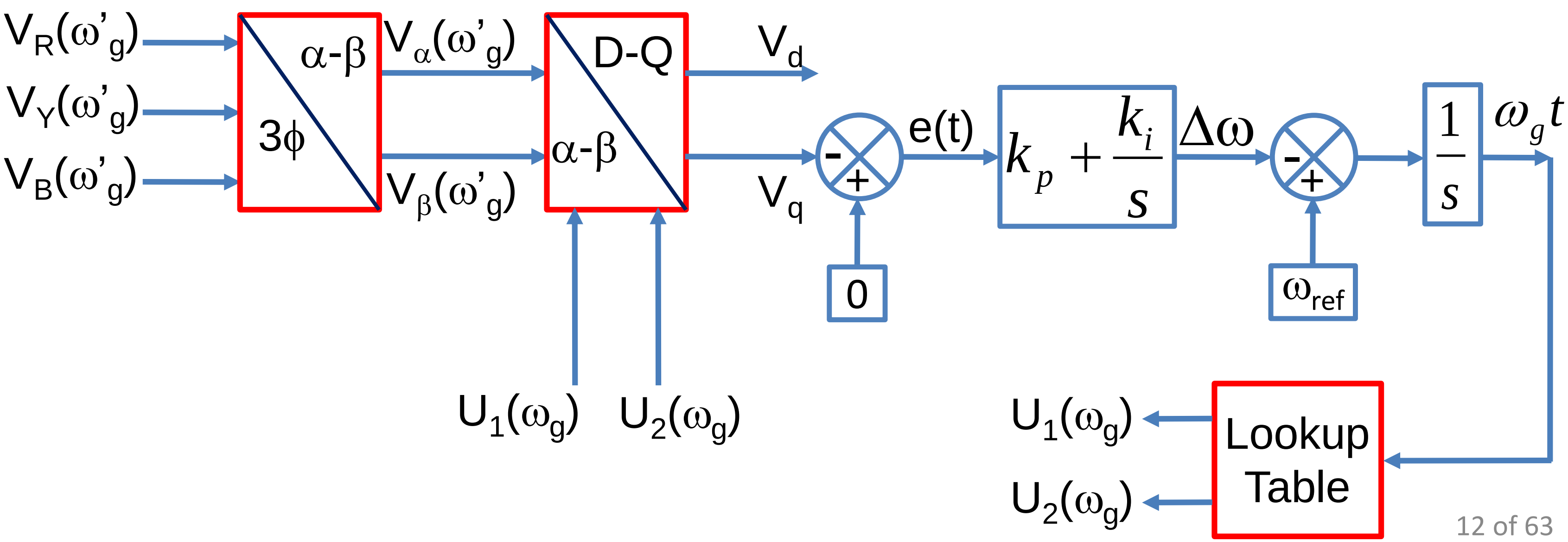
- Design of PI-controller constants ?
  - Assuming grid frequency variation is minimal, then

$$\sin((\omega_g' - \omega_g)t) \approx (\omega_g' - \omega_g)t$$

  - Hence $V_q$ is,

$$V_q = \frac{3}{2} V_g (\omega_g' - \omega_g)t$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \frac{3}{2} V_g \cos((\omega_g' - \omega_g)t) \\ \frac{3}{2} V_g \sin((\omega_g' - \omega_g)t) \end{bmatrix}$$



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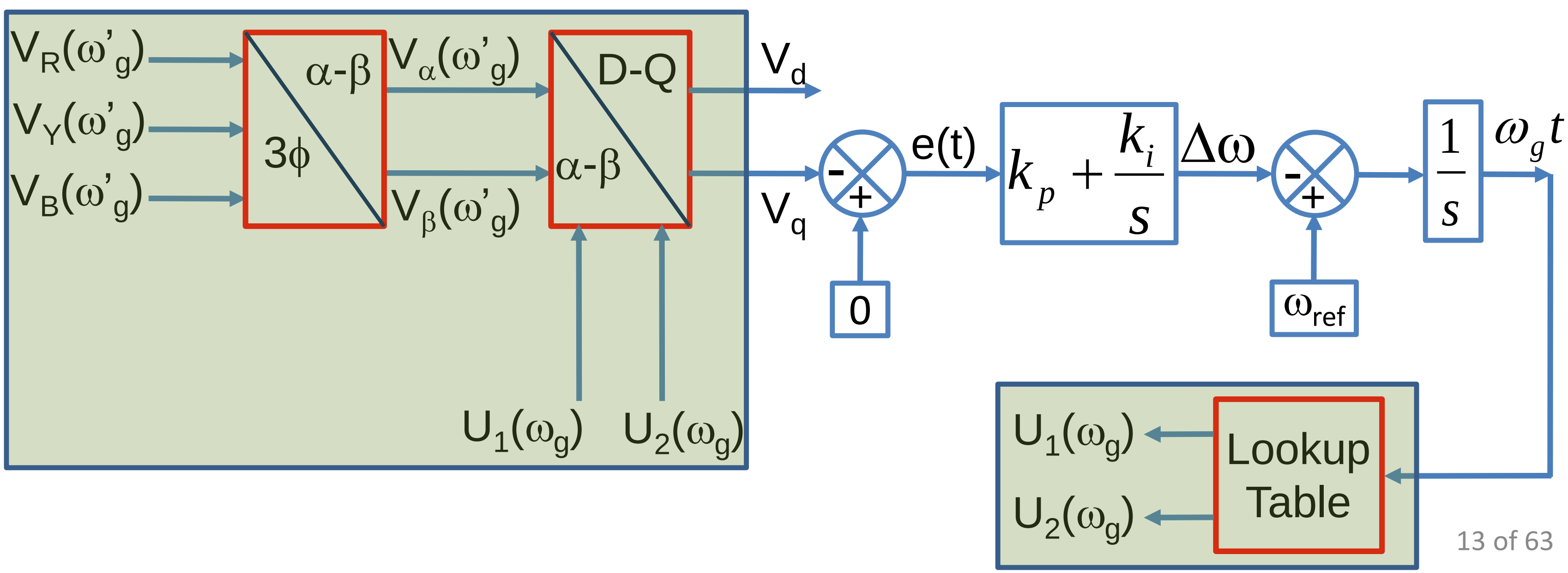

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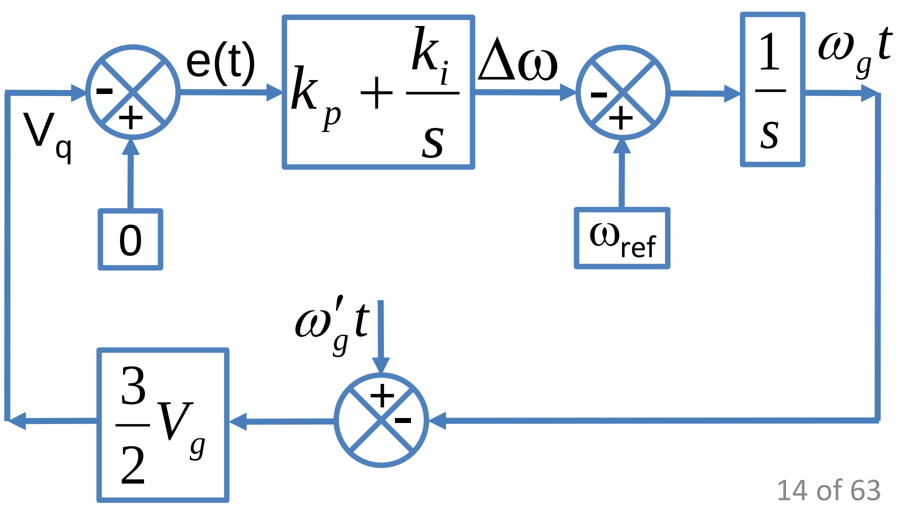
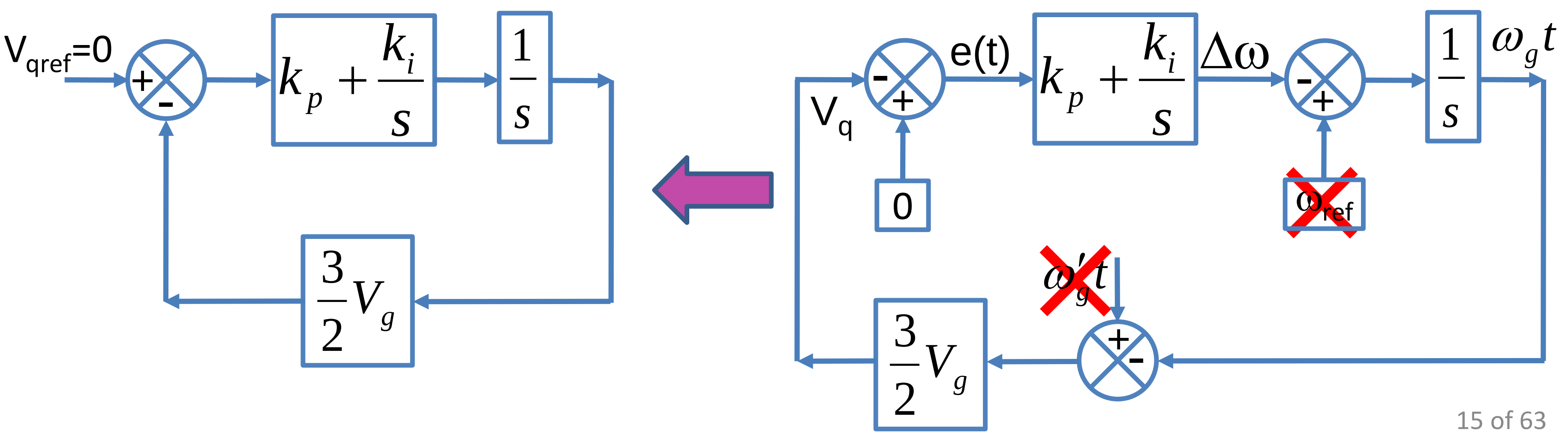


# Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants ?
  - The block diagram can be further reduced by removing the quantities not taking part in transients
  - $\omega_{ref}$ is constant and no effect on transients
  - Once grid voltage is changed from $\omega_g$ to $\omega_g'$, $\omega_g'$ can be assumed to be constant
  - Simplified block diagram is,

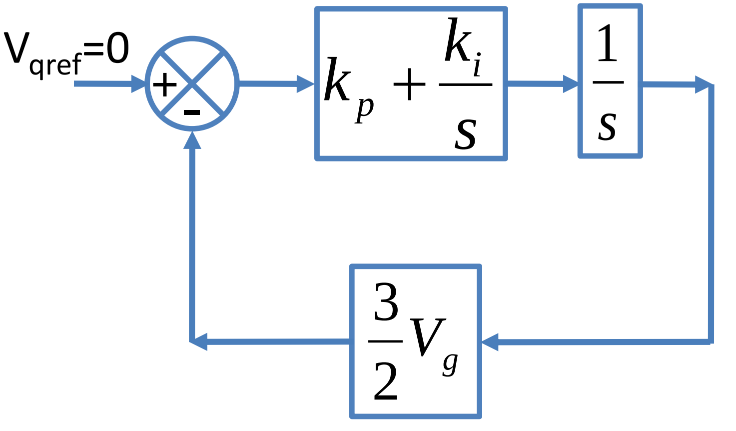


# Unit vector for Balanced/Unbalanced Grid condition

- Design of PI-controller constants ?
  - Open loop transfer function of the system is given by,

$$G(s)H(s)=\frac{3}{2}V_g\left(\frac{sk_p+k_i}{s}\right)\left(\frac{1}{s}\right)$$

$$G(s)H(s)=\frac{3}{2}V_g\left(\frac{\frac{s}{k_i/k_p}+1}{\left(s\big/\sqrt{k_i}\right)^2}\right)$$



# Unit vector for Balanced/Unbalanced Grid condition

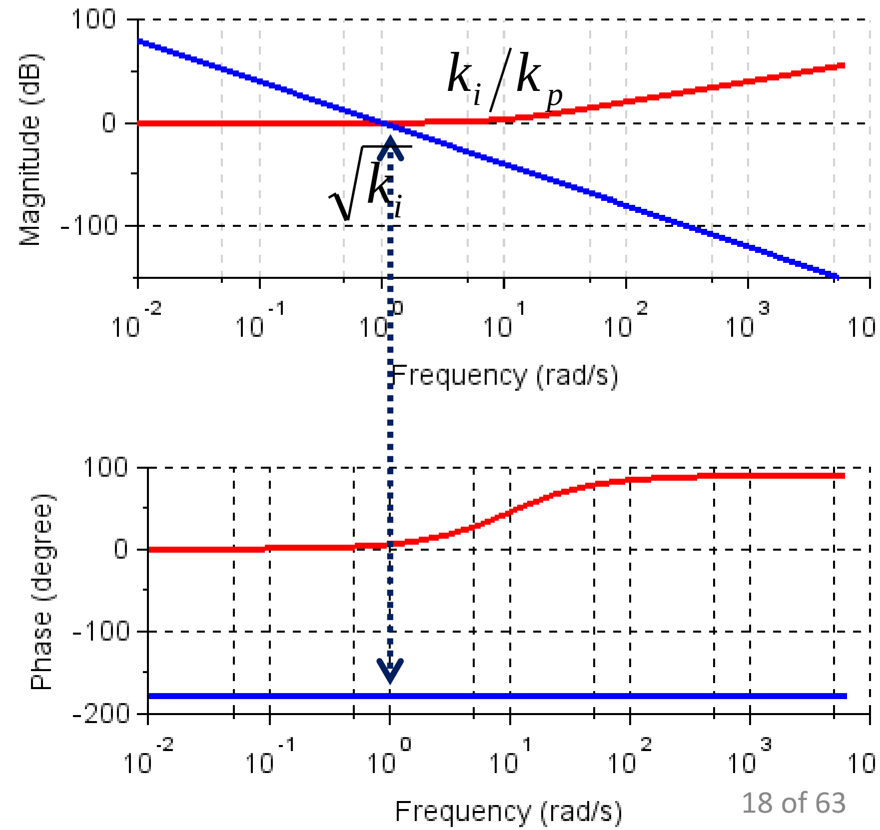
- Design of PI-controller constants ?
  - Find asymptotic bode plot of the system,
  - For zero:
    - Corner frequency, $\omega_z = k_i / k_p$
    - Magnitude plot: +20dB/decade
    - Phase plot: +45$^0$/decade
  - For poles:
    - Corner frequency, $\omega_p = \sqrt{k_i}$
    - Magnitude plot: -40dB/decade
    - Phase plot: -180$^0$

$$G(s)H(s) = \frac{3}{2} V_g \left( \frac{\dfrac{s}{k_i / k_p} + 1}{\left( s \big/ \sqrt{k_i} \right)^2} \right)$$

# Unit vector for Balanced/Unbalanced Grid condition

$$G(s)H(s) = \frac{3}{2} V_g \left( \frac{\frac{s}{k_i/k_p} + 1}{\left( \frac{s}{\sqrt{k_i}} \right)^2} \right)$$

- Design of PI-controller constants ?
  - Closed loop system will be stable,
    - If the open loop gain crosses 0dB (unity gain) with -20dB/decade and ensuring atleast $-135^0$ Phase margin
  - If $\omega_z > \omega_p$, leads to a phase close to $-180^0$ makes system prone to unstable
  - Hence to make system more stable, keep $\omega_z <= \omega_p$
  - Since phase plot of zero has a slope of $+45^0$/decade, for $\omega_z = \omega_p$, the phase is $-135^0$ leads to phase margin of $45^0$



# Unit vector for Balanced/Unbalanced Grid condition

$$G(s)H(s) = \frac{3}{2}V_g \left( \frac{\frac{s}{k_i/k_p} + 1}{\left( s/\sqrt{k_i} \right)^2} \right)$$

- Design of PI-controller constants ?
  - $\omega_z = \omega_p$ implies,

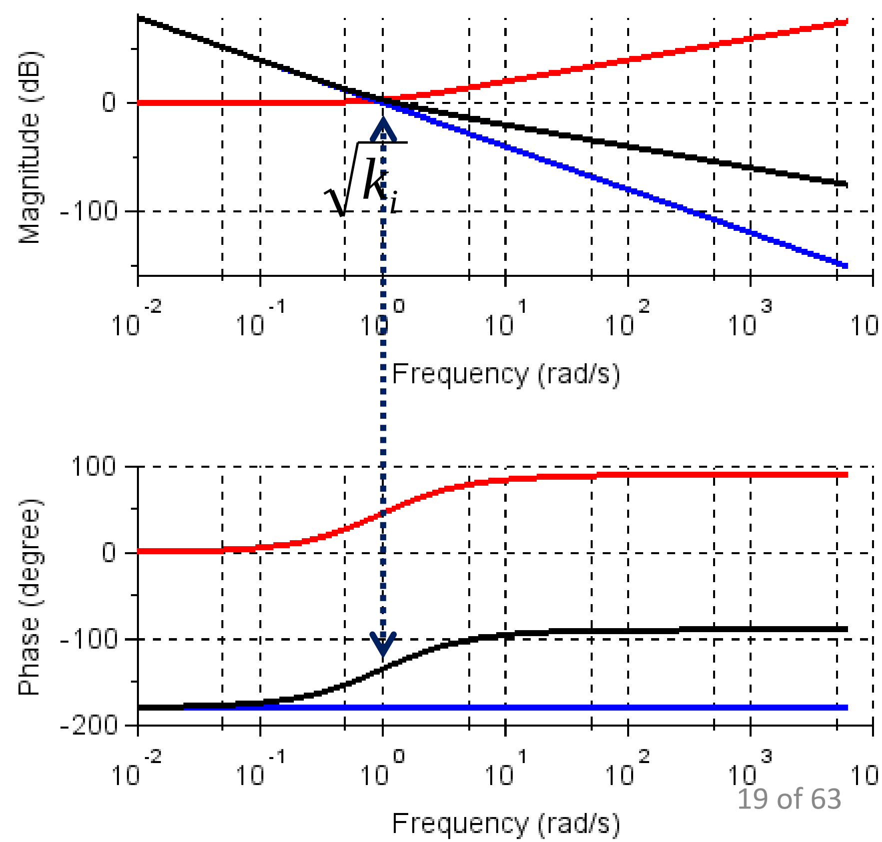
$$\frac{k_i}{k_p} = \sqrt{k_i}$$

$$k_p = \sqrt{k_i}$$

  - One more component present in the open loop transfer function is,

$$3/2\, V_g$$

  - This will not affect the phase plot, but gain cross over frequency will be pushed further away increasing the bandwidth of the system

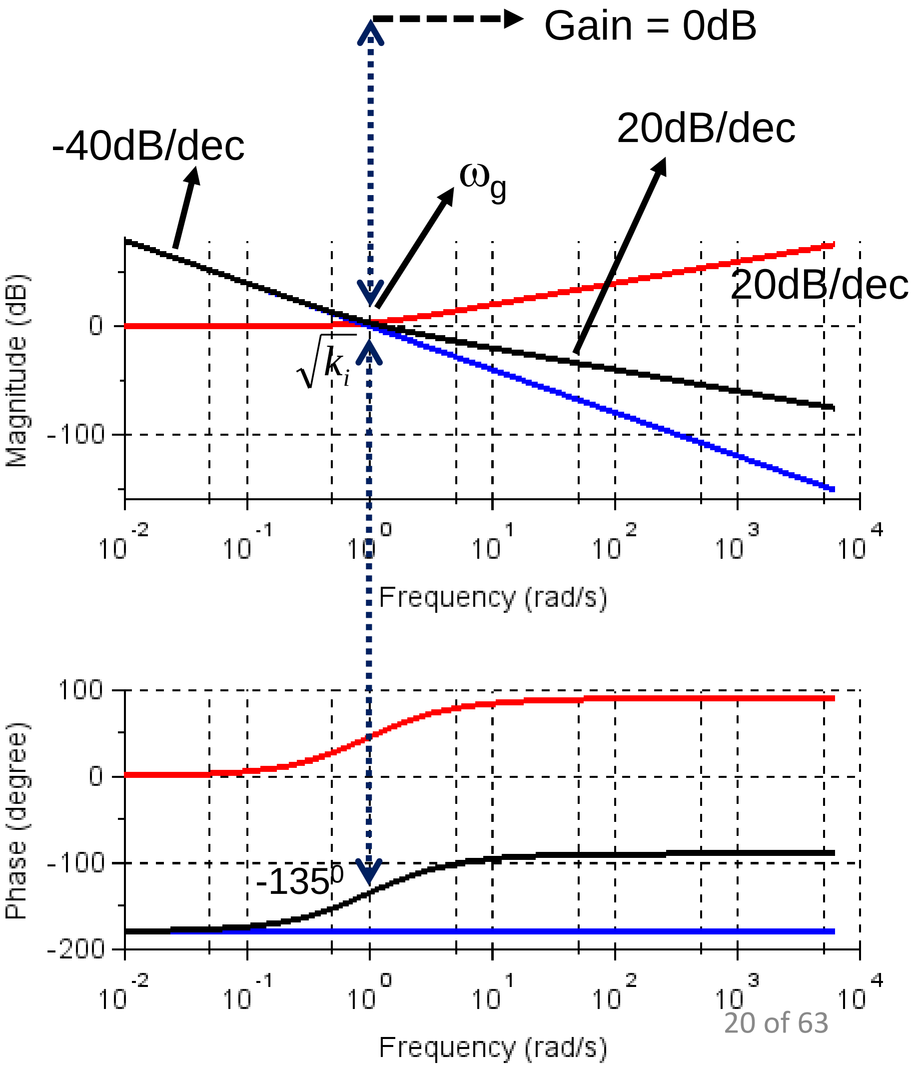


# Unit vector for Balanced/Unbalanced Grid condition

$$k_p = \sqrt{k_i}$$

- Find system bandwidth ?
  - Defined as the frequency at which the closed-loop magnitude is equal to -3 dB
  - For phase of -120$^0$, the open loop bandwidth and closed loop bandwidth are found to be closer
  - Since gain at $\sqrt{k_i}$ is 0dB, the gain cross over frequency of open loop system is,

$$\omega_g = \sqrt{k_i}$$



# Unit vector for Balanced/Unbalanced Grid condition

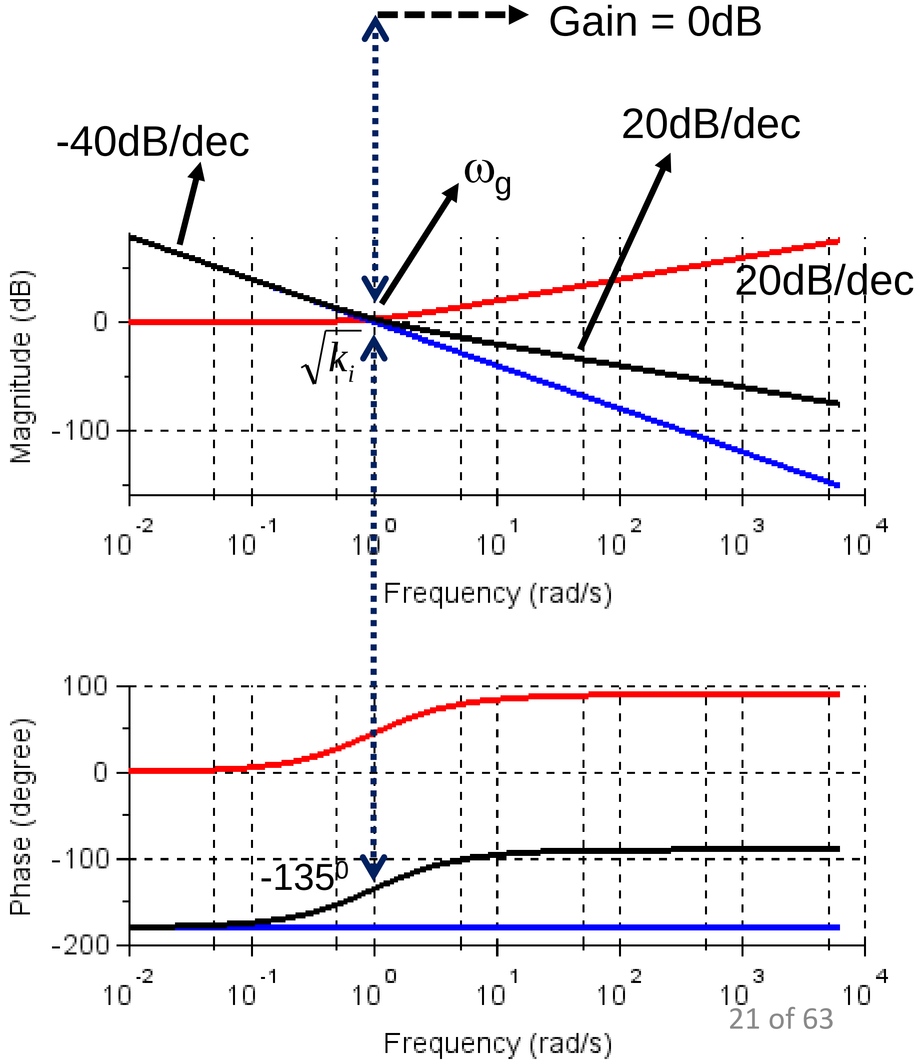
$$k_p = \sqrt{k_i}$$

- Find system bandwidth ?
  - The modified crossover frequency considering 3/2$V_g$ is,
  - Gain at $\omega_g = \sqrt{k_i}$ is,

$$20\log\left(\frac{3}{2}V_g\right)$$

  - Gain at new crossover frequency ($\omega_g'$) is 0dB
  - Slope during this time is -20dB/decade

$$\frac{0-20\log\left(\frac{3}{2}V_g\right)}{\log\omega_g' - \log\left(\sqrt{k_i}\right)} = -20$$



# Unit vector for Balanced/Unbalanced Grid condition

$$k_p = \sqrt{k_i}$$

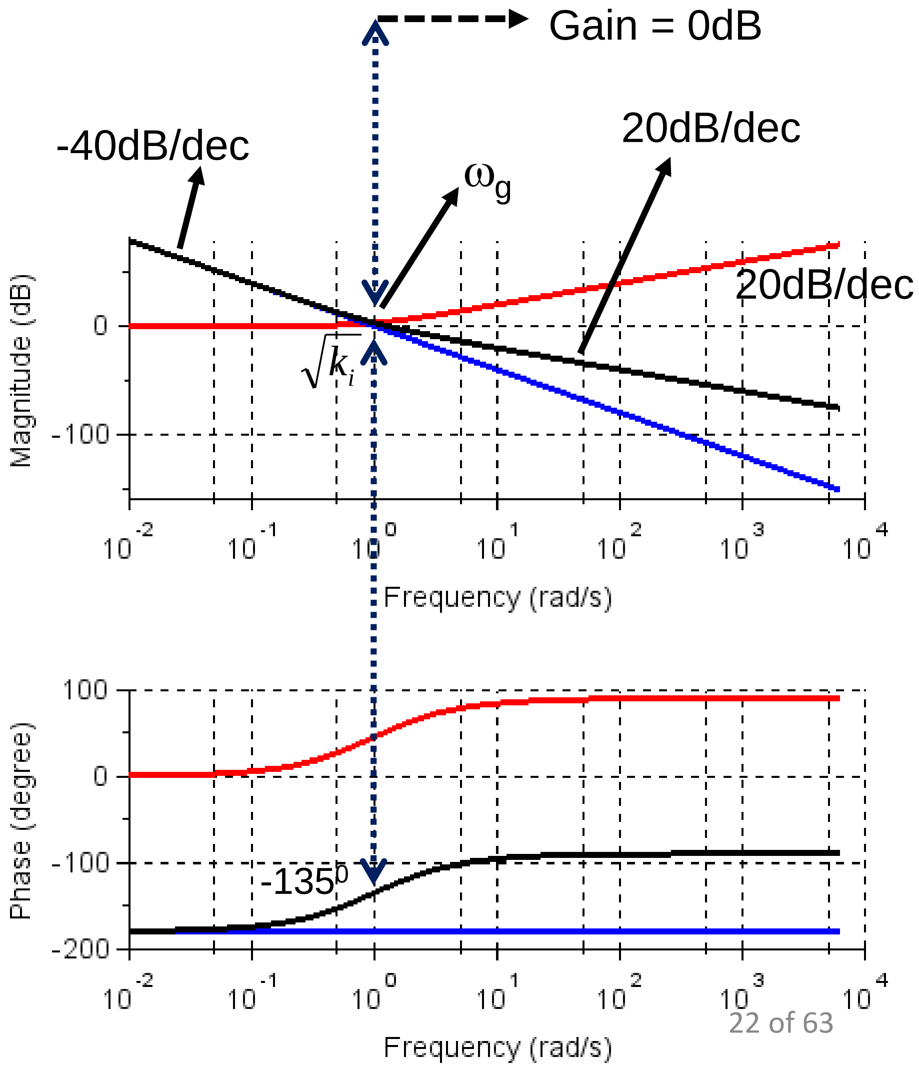
- Find system bandwidth ?
  - The modified crossover frequency considering $3/2V_g$ is,

$$\omega_g' = \frac{3}{2} V_g \sqrt{k_i}$$

  - Closed loop bandwidth (appx.) is,

$$BW = \frac{3}{2} V_g k_p$$

  - Once the bandwidth is fixed proportional constant ($k_p$) can be found



# Unit vector for Balanced/Unbalanced Grid condition

$$k_p = \sqrt{k_i}$$

- How to fix the bandwidth ?
  - Bandwidth is decided by the harmonics present in $V_d$ and $V_q$ components as well as the response time requirement in transient conditions
  - For a balanced three wire system the minimum harmonics expected on $V_d$ and $V_q$ are 300Hz (transformation of $5^{th}$ and $7^{th}$ harmonics)
    - *Here bandwidth of 30Hz to 60Hz will be sufficient for proper attenuation of harmonics in grid voltages*

- **In Summary:**
  - Based on the application and transient requirement fix the bandwidth
  - If grid voltage peak is known, proportional constant ($k_p$) can be found

$$k_p = \frac{BW}{\frac{3}{2} V_g}$$

  - Once $k_p$ is known integral constant $k_i$ can be computed

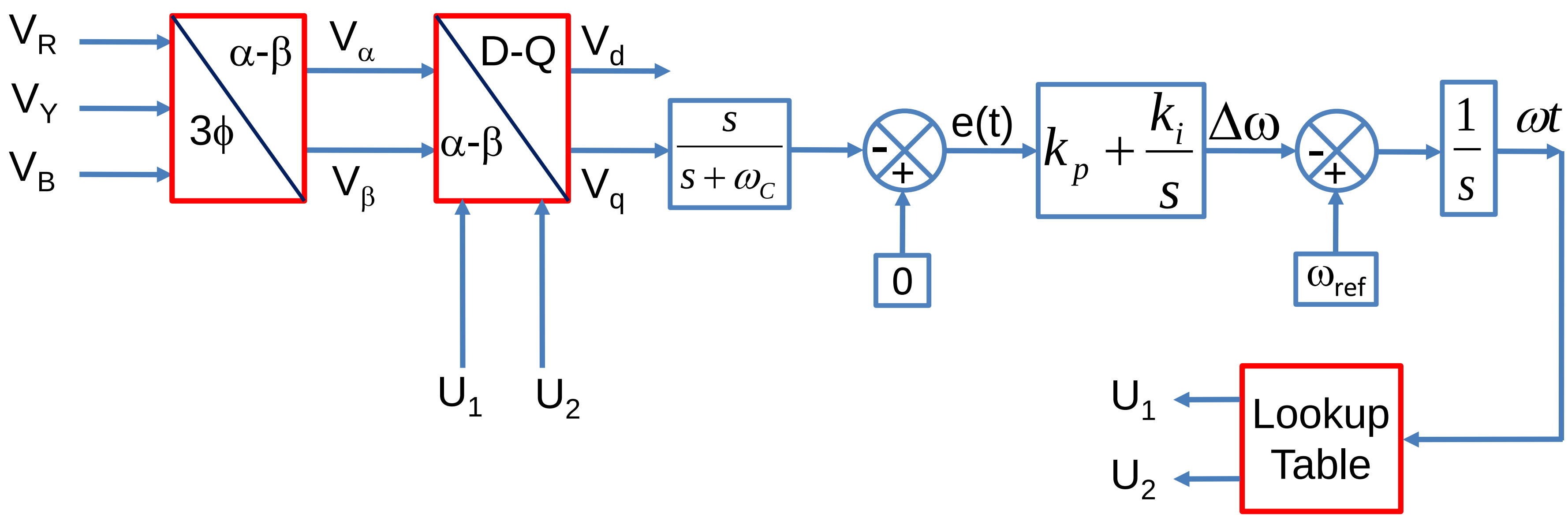
$$k_i = k_p^{\,2}$$

# Unit vector for Balanced/Unbalanced Grid condition

- Unit vector under unbalanced grid condition ?
  - Under unbalanced grid conditions, the grid voltage contains fundamental positive sequence as well as fundamental negative sequence components
  - Let us construct the unit vector such that it is **synchronized with fundamental positive sequence component**
  - As earlier fundamental positive sequence present in the grid voltage when transformed to D-Q reference frame reflected as d.c. component and fundamental negative sequence present in the grid voltage reflected as 100Hz component
  - Since we required only d.c. component in $V_q$, 100Hz component can be easily removed by using simple low pass filter of corner frequency around 10Hz

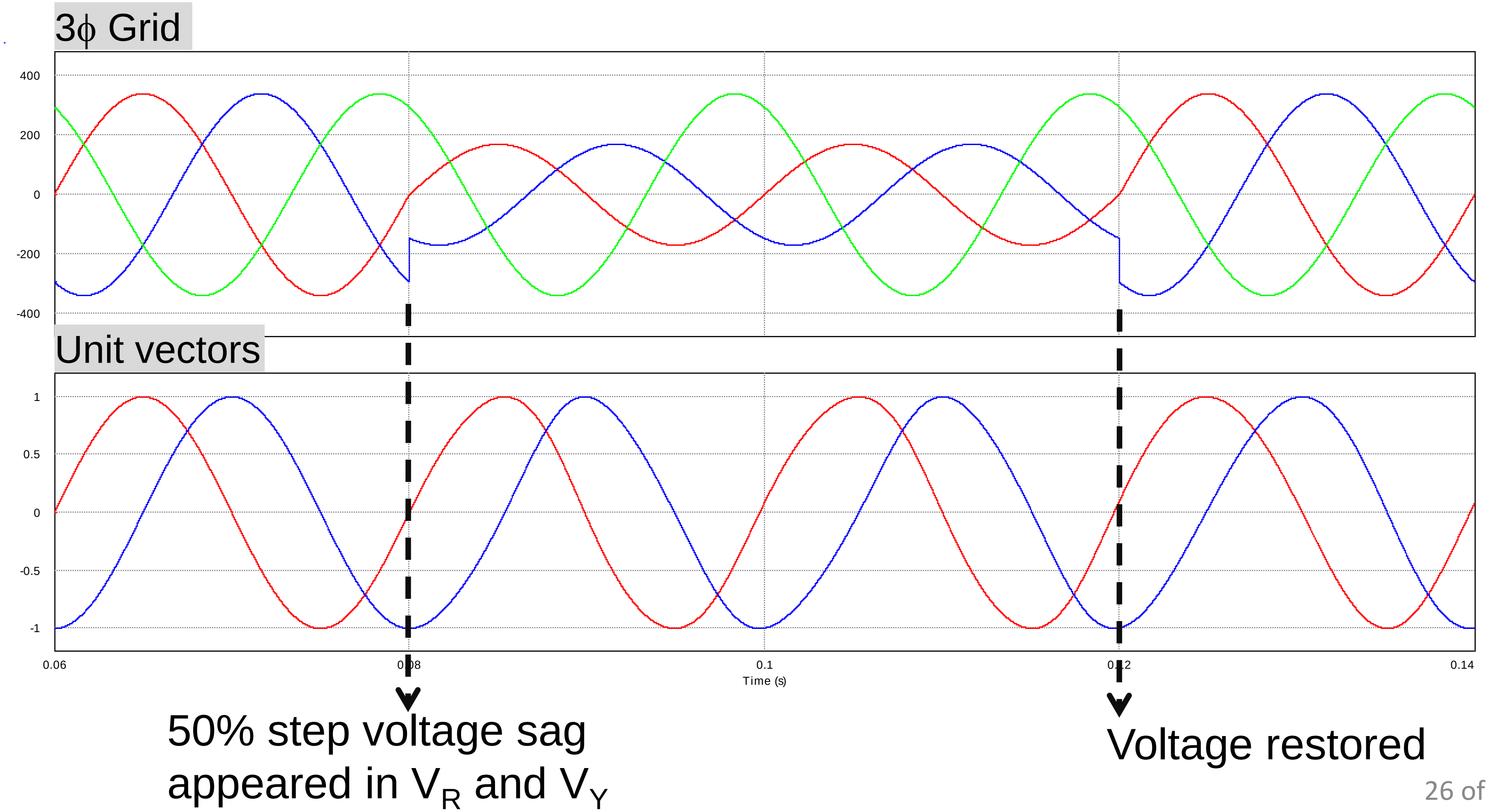
# Unit vector for Balanced/Unbalanced Grid condition

- Unit vector under unbalanced grid condition ?
  - $\omega_C$ is the corner frequency of low pass filter
  - Low pass filter will not affect the information of fundamental positive sequence, as it is a d.c. quantity



# Unit vector for Balanced/Unbalanced Grid condition

- Test results
  - Transient performance analysis carried out by simulation using PSIM simulation package



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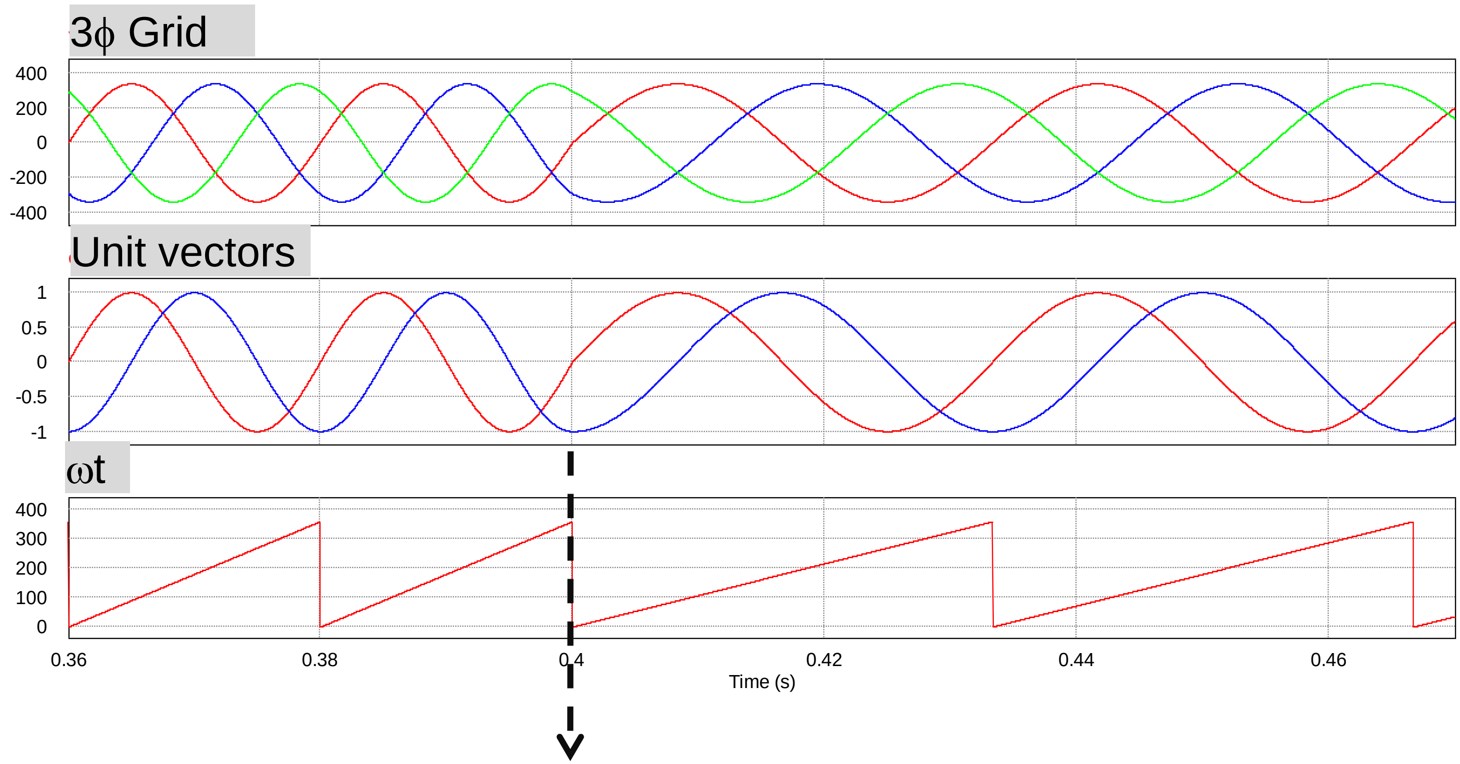
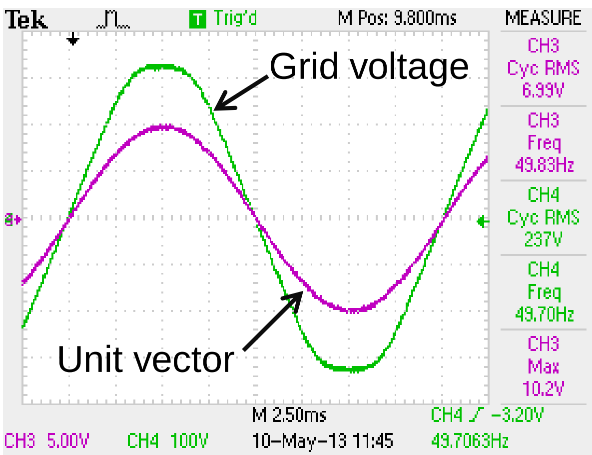


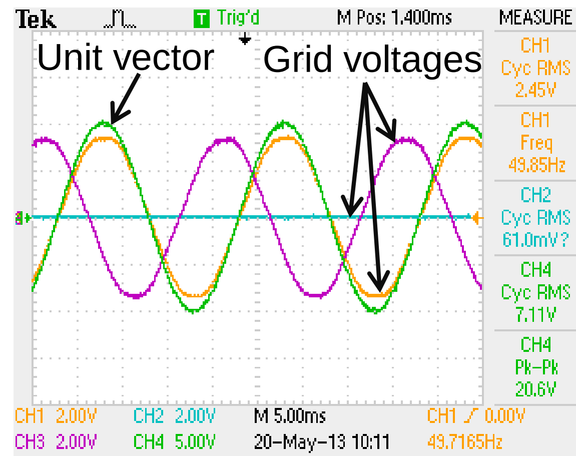

Step change in frequency (from 50Hz to 30Hz) appeared in $V_R$, $V_Y$ and $V_B$

# Unit vector for Balanced/Unbalanced Grid condition

- Test results
  - Unit vector implemented for unbalanced grid condition in TI's DSP TMS320F2812

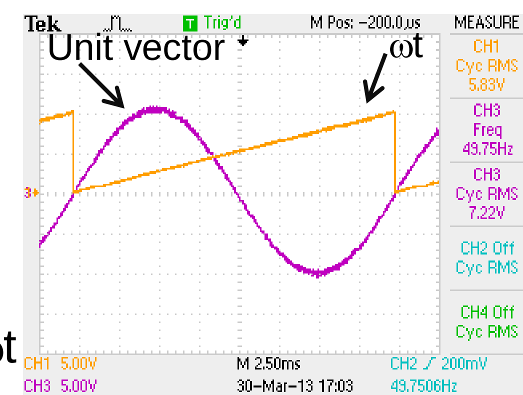



Balanced grid voltage




$V_R$ and $V_B$ are nominal, $V_Y$=0

Unit vector and angle $\omega t$



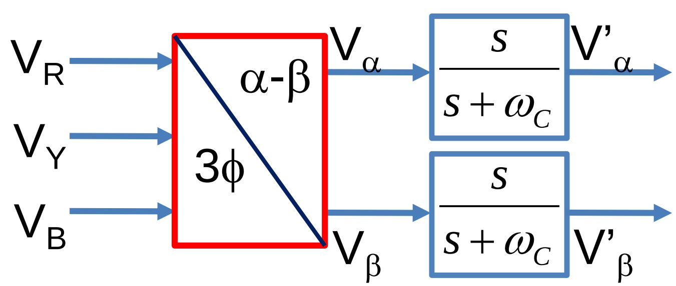
# A Simple method for Unit vector in Balanced Grid condition

- A simple method exists based on trigonometry to compute unit vector for balanced grid condition
- Transform three phase grid voltage to α-β co-ordinate

$$V_\alpha(t) = \frac{3}{2} V_g \sin \omega_g t, \quad V_\beta(t) = -\frac{3}{2} V_g \cos \omega_g t$$

- The output of Low Pass Filter (LPF) is given by,

$$V'_\alpha(s) = \frac{s}{s+\omega_c} V_\alpha(s), \quad V'_\beta(s) = \frac{s}{s+\omega_c} V_\beta(s)$$



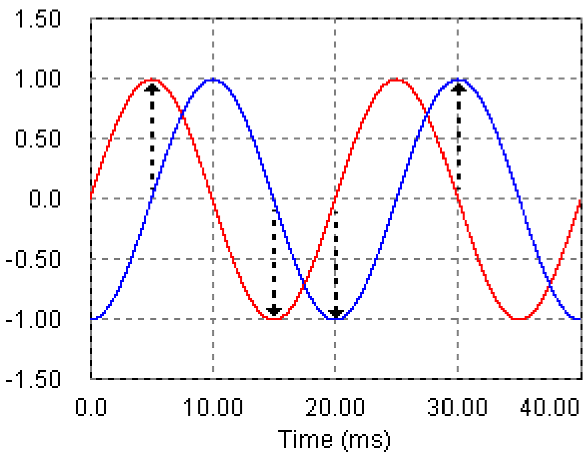
# A Simple method for Unit vector in Balanced Grid condition

- Output of LPF in time domain is,

$$V'_\alpha(t) = \frac{\tfrac{3}{2} V_g \omega_c}{\sqrt{\omega_g^2 + \omega_c^2}} \sin(\omega_g t - \phi), \quad V'_\beta(t) = -\frac{\tfrac{3}{2} V_g \omega_c}{\sqrt{\omega_g^2 + \omega_c^2}} \cos(\omega_g t - \phi)$$

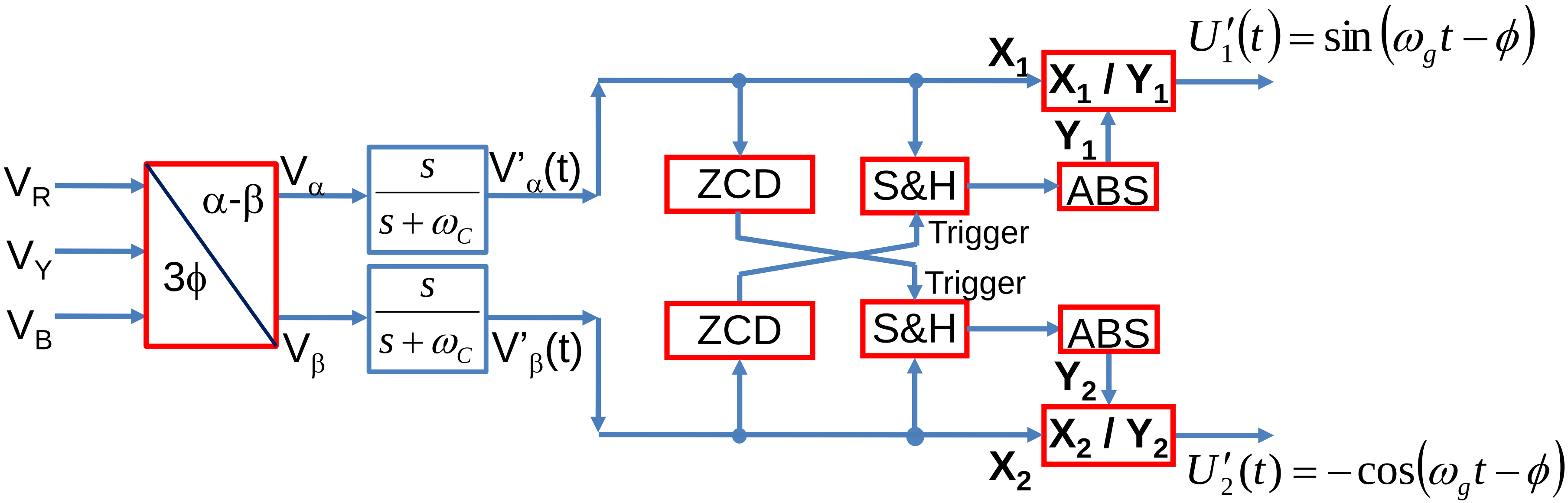
where, $\tan\phi = \dfrac{\omega_g}{\omega_c}$

- $V'_\alpha(t)$ and $V'_\beta(t)$ are always $90^0$ displaced irrespective of grid frequency and corner frequency of LPF
- At the zero crossing of $V'_\alpha(t), V'_\beta(t)$ will be in peak and vice versa
- Unit magnitude of $V'_\alpha(t)$ and $V'_\beta(t)$ can be obtained by dividing the term by its own magnitude,



# A Simple method for Unit vector in Balanced Grid condition

- Block diagram of unit vector construction



# A Simple method for Unit vector in Balanced Grid condition

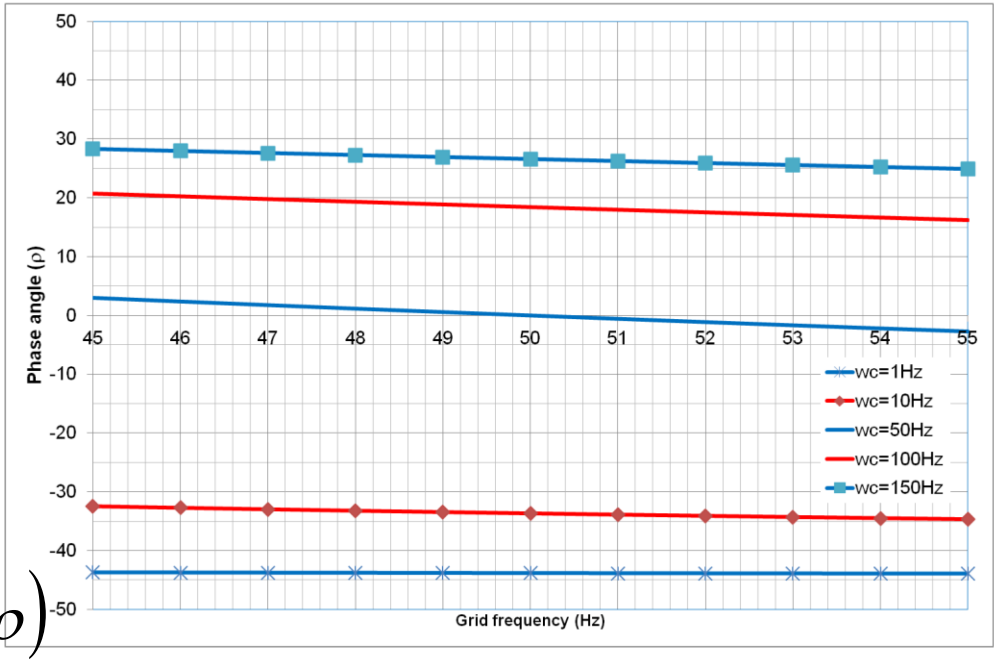
- Let us perform the following operations,

$$F_1(t) = V'_\alpha(t) - V'_\beta(t)$$

$$= \frac{\frac{3}{\sqrt{2}} V_g \omega_c}{\sqrt{\omega_c^{\,2} + \omega_g^{\,2}}} \sin\left(\omega_g t + \rho\right)$$

$$F_2(t) = V'_\alpha(t) + V'_\beta(t)$$

$$= -\frac{\frac{3}{\sqrt{2}} V_g \omega_c}{\sqrt{\omega_c^{\,2} + \omega_g^{\,2}}} \cos\left(\omega_g t + \rho\right)$$



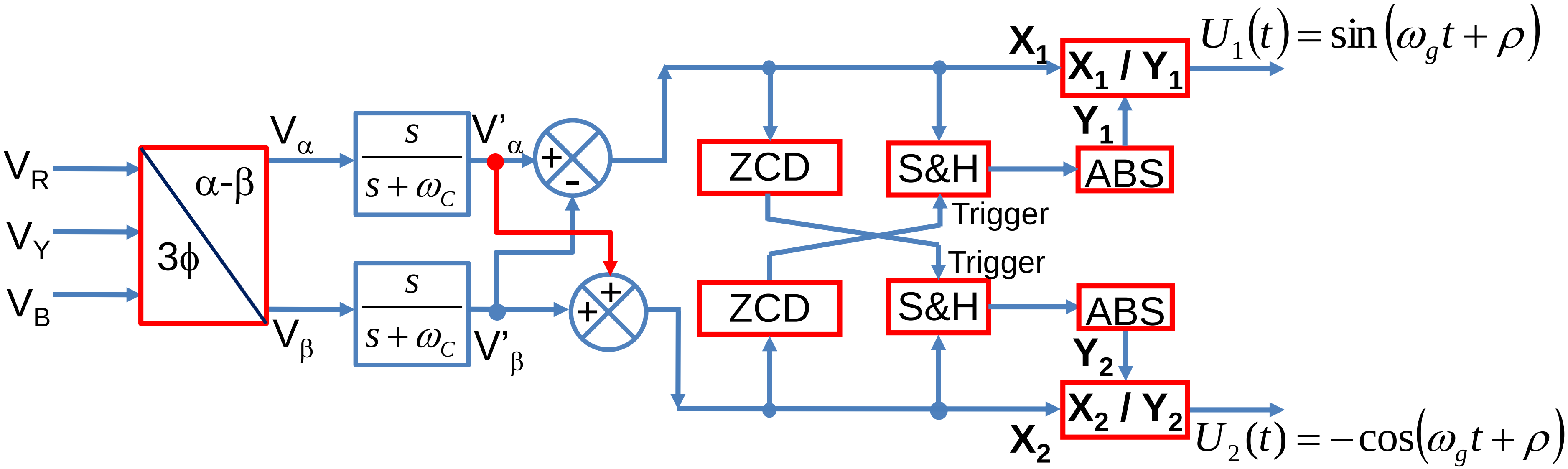

where, $\tan\rho = \dfrac{\omega_c - \omega_g}{\omega_c + \omega_g}$

- Phase angle (ρ) is minimum when $\omega_c = 2\pi 50$
- For $\omega_c = 2\pi 50$, $\rho$ is zero when grid frequency is 50Hz

# A Simple method for Unit vector in Balanced Grid condition

- Complete block diagram of unit vector construction

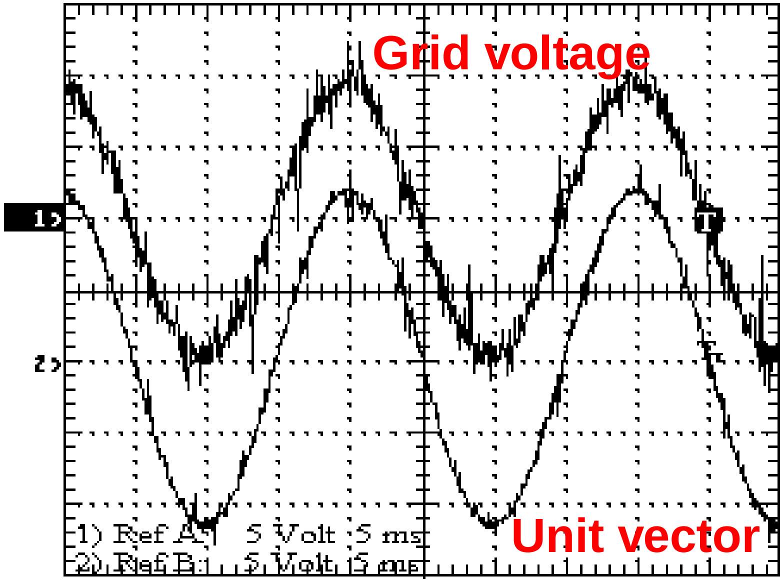
**where, $\omega_c = 2\pi 50$**




- This method will introduce a small phase angle error when grid frequency varies
- Each harmonics is reduced by $\sqrt{\frac{2}{1+h^2}}$ when compared to fundamental

# A Simple method for Unit vector in Balanced Grid condition

- Test results



# SESSION 2

## UNIT VECTORS FOR SINGLE PHASE GRID

# Overview of the presentation

- Constructing two unit magnitude $90^0$ displaced components
- Mitigating the effect of grid frequency variation
  - ✓Approximation method
  - ✓Rigorous method

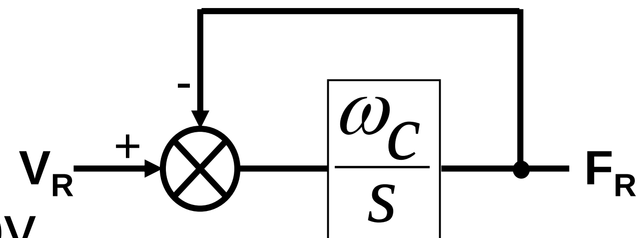
# Overview of the presentation

- Constructing two unit magnitude $90^0$ displaced components
- Mitigating the effect of grid frequency variation
  - ✓ Approximation method
  - ✓ Rigorous method

# Innovative method for constructing unit vector

- Let R-phase grid voltage be, $V_R = V_g \sin(\omega_S t)$
- Let the above voltage is passed through a LPF of corner frequency, $\omega_c$
- Using Laplace analysis

$$F_R(s) = \frac{V_g \omega_C \omega_S}{\omega_S^2 + \omega_C^2}\left(\frac{1}{s+\omega_C} + \left(\frac{\omega_C}{\omega_S}\right)\frac{\omega_S}{s^2+\omega_S^2} - \frac{s}{s^2+\omega_S^2}\right)$$




- The steady state output, $F_R(t)$ is given by

$$F_R(t)|_{steady} = \frac{V_g \omega_C}{\sqrt{\omega_S^2 + \omega_C^2}} \sin(\omega_S t - \phi) \quad \text{and} \quad \tan\phi = \frac{\omega_S}{\omega_C}$$

- Transient term of $F_R(t)$ is given by,

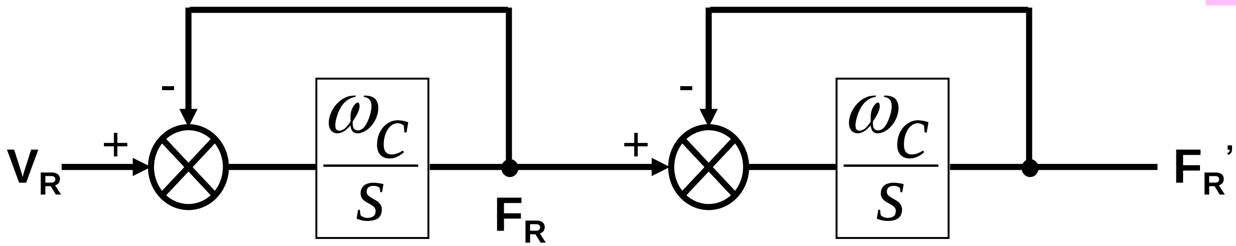
$$F_R(t)|_{transient} = \frac{V_g \omega_C \omega_S}{\left(\omega_C^2 + \omega_S^2\right)} e^{-\omega_C t}$$

# constructing unit vector (contd.)

$F_R(t)|_{steady} = \frac{V_g\omega_C}{\sqrt{\omega_S^2+\omega_C^2}}\sin(\omega_S t-\phi)$

$V_R = V_g \sin(\omega_S t)$

- Let the $F_R(t)$ is again pass through a LPF of same corner frequency, $\omega_c$




- Using Laplace analysis

$$F_R'(s)=V_g\omega_C^2\omega_S\frac{1}{\left(\omega_S^2+\omega_C^2\right)^2}\left(2\omega_C\frac{1}{s+\omega_C}+\left(\omega_S^2+\omega_C^2\right)\frac{1}{(s+\omega_C)^2}-2\omega_C\frac{s}{s^2+\omega_S^2}+\left(\omega_C^2-\omega_S^2\right)\frac{1}{s^2+\omega_S^2}\right)$$

- The steady state output, $F_R'(t)$ is given by

$$F_R'(t)=-\frac{V_g\omega_C^2}{\left(\omega_S^2+\omega_C^2\right)}\cos(\omega_S t+\psi) \quad \text{and} \quad \cos\psi=\frac{2\omega_C\omega_S}{\left(\omega_S^2+\omega_C^2\right)}, \quad \sin\psi=\frac{\left(\omega_C^2-\omega_S^2\right)}{\left(\omega_S^2+\omega_C^2\right)}$$

- Transient term of $F_R'(t)$ is given by,

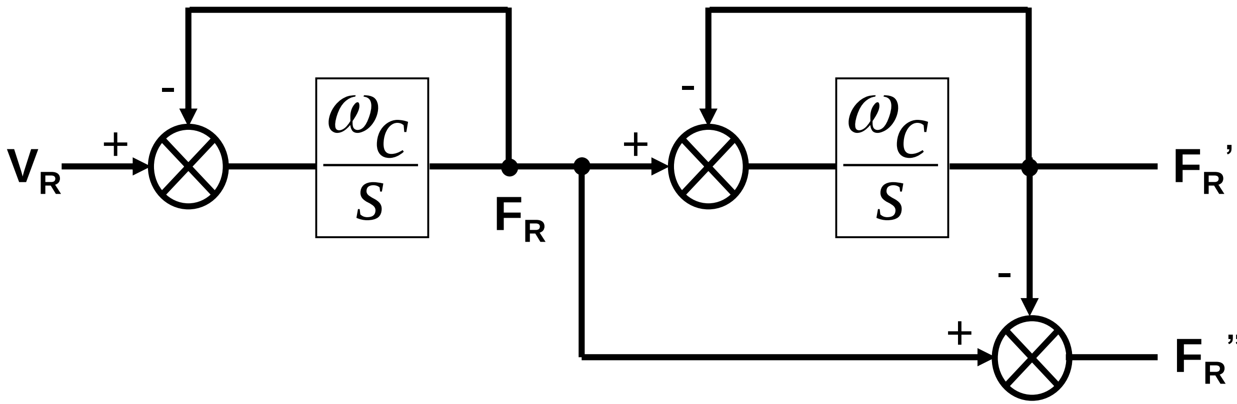
$$F_R'(t)|_{transient}=\frac{V_g\omega_C^2\omega_S}{\left(\omega_C^2+\omega_S^2\right)^2}\left(2\omega_C+\left(\omega_C^2+\omega_S^2\right)t\right)e^{-\omega_C t}$$

# constructing unit vector (contd.)

$$F_R'(t) = -\frac{V_g \omega_C^2}{(\omega_S^2 + \omega_C^2)} \cos(\omega_S t + \psi)$$

$V_R = V_g \sin(\omega_S t)$

- Subtract $F_R'(t)$ from $F_R(t)$, let the result be $F_R''(t)$




- The steady state output, $F_R''(t)$ is given by

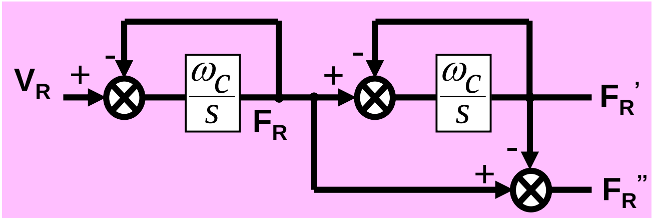
$$F_R''(t) = \frac{V_g \omega_C \omega_S}{(\omega_S^2 + \omega_C^2)} \sin(\omega_S t + \psi) \quad \text{and} \qquad \cos\psi = \frac{2\omega_C \omega_S}{(\omega_S^2 + \omega_C^2)}, \quad \sin\psi = \frac{(\omega_C^2 - \omega_S^2)}{(\omega_S^2 + \omega_C^2)}$$

- Transient term of $F_R''(t)$ is given by,

$$F_R''(t)|_{transient} = \frac{V_g \omega_C \omega_S}{(\omega_C^2 + \omega_S^2)} \left( \frac{(\omega_S^2 - \omega_C^2)}{(\omega_C^2 + \omega_S^2)} + \omega_C t \right) e^{-\omega_C t}$$

# constructing unit vector (contd.)

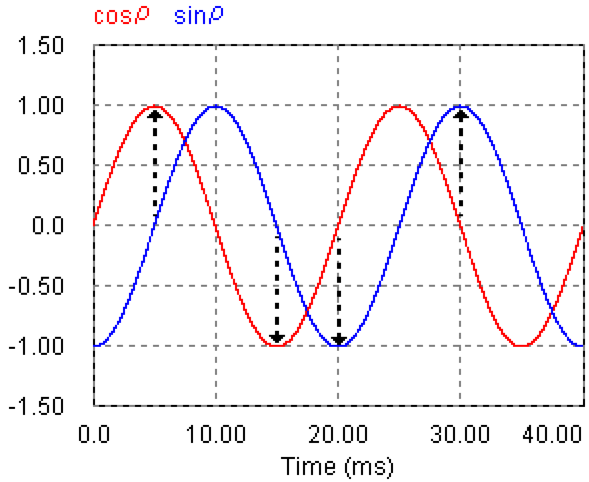
- Comparing steady state term of $F_R'(t)$ and $F_R''(t)$



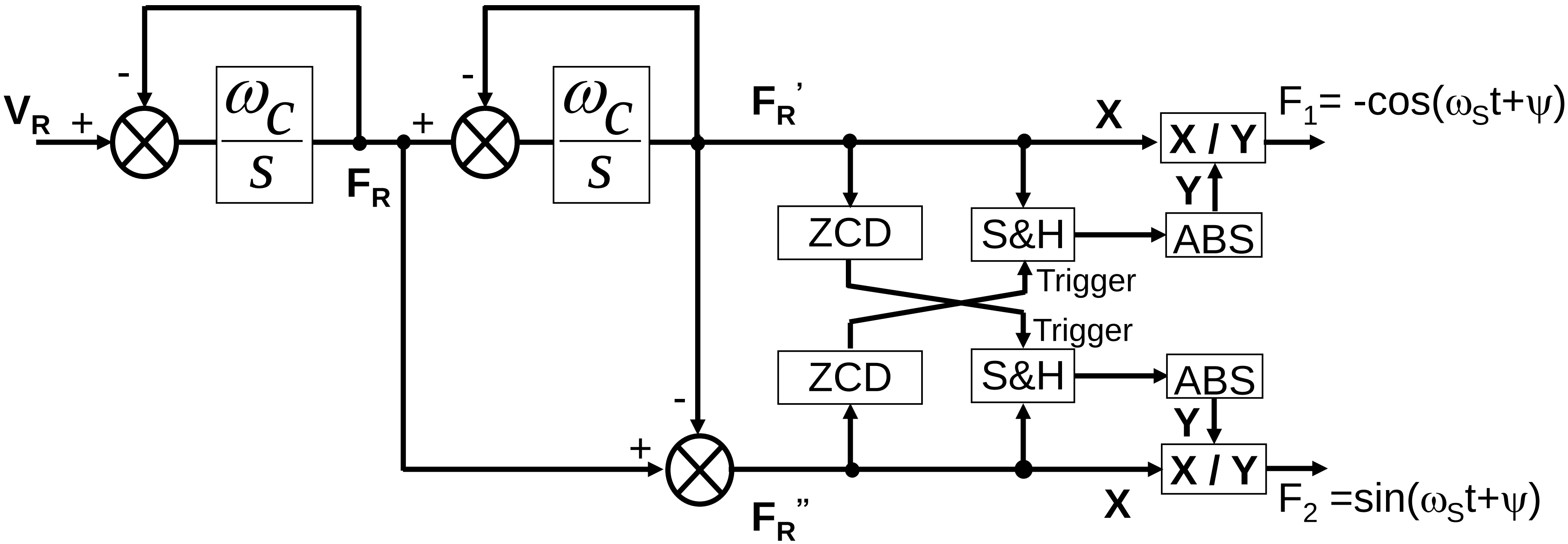

$$F_R'(t) = -\frac{V_g \omega_C^2}{(\omega_S^2 + \omega_C^2)} \cos(\omega_S t + \psi)$$

$$F_R''(t) = \frac{V_g \omega_C \omega_S}{(\omega_S^2 + \omega_C^2)} \sin(\omega_S t + \psi)$$

- $F_R'(t)$ and $F_R''(t)$ are always $90^0$ displaced irrespective of grid frequency and corner frequency of LPF
- At the zero crossing of $F_R''(t)$, $F_R'(t)$ will be in peak and vice versa
- Unit magnitude of $F_R'(t)$ and $F_R''(t)$ can be obtained by dividing the term by its own magnitude



# constructing unit vector (contd.)




- Inference from the above result
  - ✓ $F_2(t)$ is phase shifted from grid voltage by an angle $\psi$

$V_R = V_g \sin(\omega_S t)$

# constructing unit vector (contd.)

- Inference from the above result (contd.)
  - ✓ If corner frequency of LPF ($\omega_c$) is set equal to grid frequency ($\omega_s$), i.e. $\omega_c = \omega_s$ :
    - ❖ phase shift $\psi = 0$
    - ❖ $F_2(t)$ is in phase with grid voltage and $F_1(t)$ is lagging the grid voltage by $90^0$

$V_R = V_g \sin(\omega_S t)$

$F_1(t) = -\cos(\omega_S t+\psi)$

$F_2(t) = \sin(\omega_S t+\psi)$

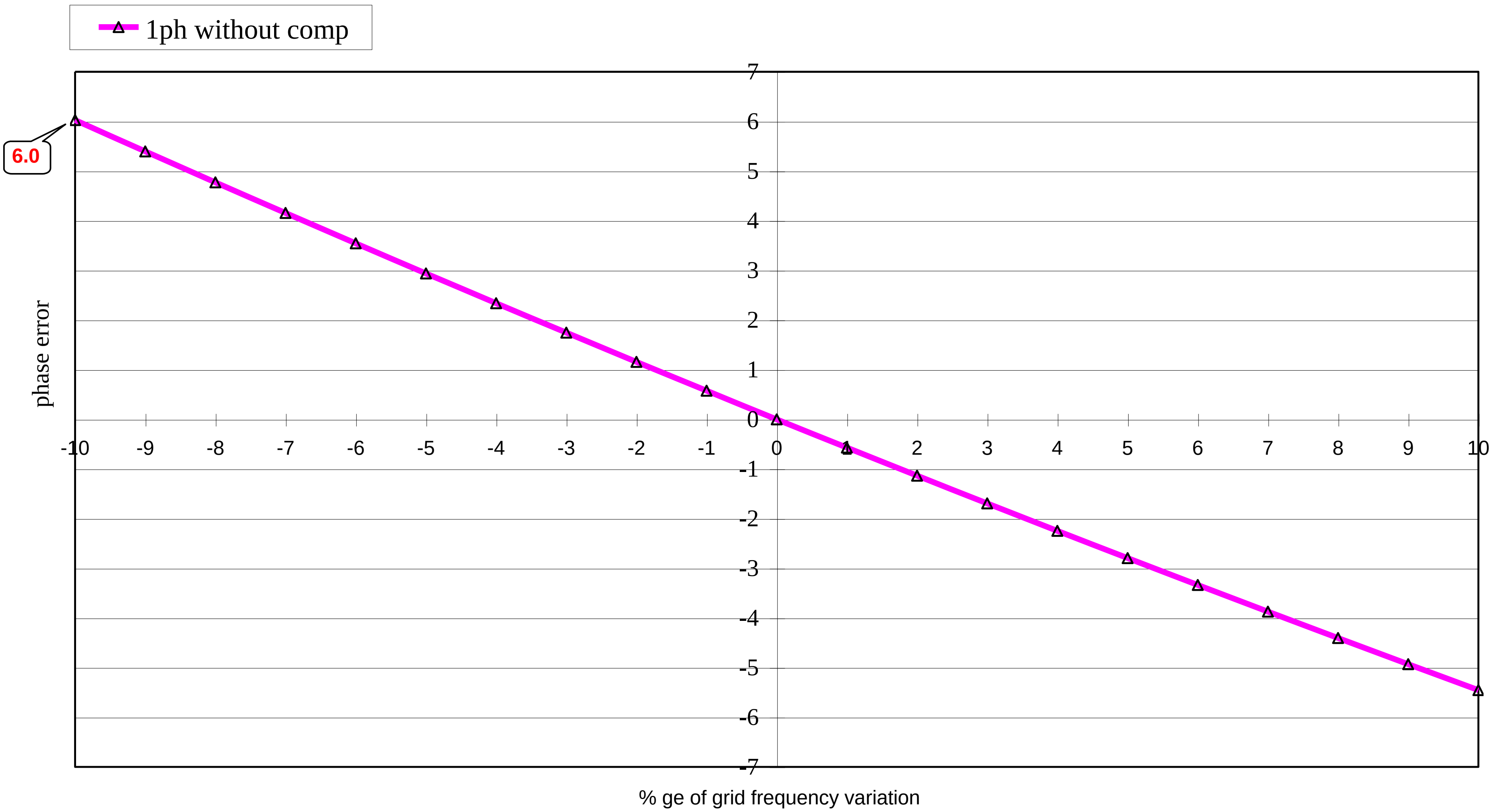
$$\sin\psi = \frac{\left(\omega_C^{\,2} - \omega_S^{\,2}\right)}{\left(\omega_S^{\,2} + \omega_C^{\,2}\right)}$$

$$\cos\psi = \frac{2\omega_C\omega_S}{\left(\omega_S^{\,2} + \omega_C^{\,2}\right)}$$

- Fix the corner frequency of LPF ($\omega_c$) is equal to $\omega_s = 2\pi 50$ rad/sec, where grid frequency $f_s$ = 50Hz
- Let Grid frequency varies a maximum of $\pm$10% (45Hz to 55Hz)
- Phase shift, $\psi = 6^0$ for -10% of grid frequency variation
- Loose the proper synchronization

# constructing unit vector (contd.)

- Phase error with variation in grid frequency



# Overview of the presentation

- Constructing two unit magnitude 900 displaced components
- Mitigating the effect of grid frequency variation
  - ✓ Approximation method
  - ✓ Rigorous method

# constructing unit vector (contd.)

## ➤ Mitigating the effect of grid frequency variation

- ✓ Give a $\Delta\omega$ variation for the grid frequency $\omega_s$ in the peak of $F_R''(t)$
- ✓ Substitute $\omega_c = \omega_s$ and solving will give

$V_R = V_g \sin(\omega_S t)$

$$F_R''(t) = \frac{V_g \omega_C \omega_S}{(\omega_S^2 + \omega_C^2)} \sin(\omega_S t + \psi)$$

$$F_R''\Big|_{\substack{\omega_S = \omega_S + \Delta\omega \\ \omega_C = \omega_S}} = \frac{V_g}{2} \frac{\left(1 + \left(\frac{\Delta\omega}{\omega_S}\right)\right)}{\left(1 + \left(\frac{\Delta\omega}{\omega_S}\right) + \frac{1}{2}\left(\frac{\Delta\omega}{\omega_S}\right)^2\right)}$$

$$F_R''\Big|_{\substack{\omega_S = \omega_S + \Delta\omega \\ \omega_C = \omega_S}} \cong \frac{V_g}{2} \quad \text{if } \frac{1}{2}\left(\frac{\Delta\omega}{\omega_S}\right)^2 \text{ is negligible compare to } 1$$

- ✓ Peak of $F_R''(t)$, $F''_{R(peak)}$, is more or less independent of grid frequency variation

$$F''_{R(peak)} \cong V_g/2 \qquad \text{for } \omega_c = \omega_s$$

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- The following trignometric relation can be used to eliminate the phase angle ψ

$$\sin(\omega_S t+\psi)V_g\cos\psi-\cos(\omega_S t+\psi)V_g\sin\psi=V_g\sin\omega_S t$$

$$-\cos(\omega_S t+\psi)V_g\cos\psi-\sin(\omega_S t+\psi)V_g\sin\psi=-V_g\cos\omega_S t$$

- $V_g\cos\psi = 2F''_{R(peak)}$

- Solve the following mathematical relation

$$2F'_{R(peak)}-V_g=\frac{2V_g\omega_C^{\ 2}}{(\omega_S^{\ 2}+\omega_C^{\ 2})}-V_g=V_g\frac{(\omega_C^{\ 2}-\omega_S^{\ 2})}{(\omega_S^{\ 2}+\omega_C^{\ 2})}$$

$$2F'_{R(peak)}-V_g=V_g\sin\psi$$

$$V_g\sin\psi\cong 2F'_{R(peak)}-2F''_{R(peak)}$$

$V_R = V_g\sin(\omega_S t)$

$F_1(t) = -\cos(\omega_S t+\psi)$

$F_2(t) = \sin(\omega_S t+\psi)$

$$\sin\psi=\frac{(\omega_C^{\ 2}-\omega_S^{\ 2})}{(\omega_S^{\ 2}+\omega_C^{\ 2})}$$

$$\cos\psi=\frac{2\omega_C\omega_S}{(\omega_S^{\ 2}+\omega_C^{\ 2})}$$

$$F_R''(t)=\frac{V_g\omega_C\omega_S}{(\omega_S^{\ 2}+\omega_C^{\ 2})}\sin(\omega_S t+\psi)$$

$$F''_{R(peak)}=\frac{V_g\omega_C\omega_S}{(\omega_C^{2}+\omega_S^{2})}$$

$F''_{R(peak)} \cong V_g/2$ for $\omega_c = \omega_S$

$$F_R'(t)=-\frac{V_g\omega_C^{\ 2}}{(\omega_S^{\ 2}+\omega_C^{\ 2})}\cos(\omega_S t+\psi)$$

$$F'_{R(peak)}=\frac{V_g\omega_C^{2}}{(\omega_C^{2}+\omega_S^{2})}$$

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

$V_R = V_g \sin(\omega_S t)$

$$F'_{R(peak)} = \frac{V_g \omega_C^2}{(\omega_C^2 + \omega_S^2)}$$

$$F''_{R(peak)} = \frac{V_g \omega_C \omega_S}{(\omega_C^2 + \omega_S^2)}$$

$$F_R''(t) = \frac{V_g \omega_C \omega_S}{(\omega_S^2 + \omega_C^2)} \sin(\omega_S t + \psi)$$

$$F_R'(t) = -\frac{V_g \omega_C^2}{(\omega_S^2 + \omega_C^2)} \cos(\omega_S t + \psi)$$

- ✓ Rewriting the relation

$$\sin(\omega_S t + \psi) V_g \cos\psi - \cos(\omega_S t + \psi) V_g \sin\psi = V_g \sin \omega_S t$$

- ➢ Substituting
  - ✓ $V_g \cos\psi = 2F''_{R(peak)}$
  - ✓ $V_g \sin\psi \cong 2F'_{R(peak)} - 2F''_{R(peak)}$

$$V_g \sin \omega_S t \cong U_1 = \sin(\omega_S t + \psi)(2F''_{R(peak)}) - \cos(\omega_S t + \psi)(2F'_{R(peak)} - 2F''_{R(peak)})$$

$$U_1 = 2F_R''(t) + 2F_R'(t) + 2\left[F''_{R(peak)} \cos(\omega_S t + \psi)\right]$$

$$U_1 = \frac{2V_g \omega_C \sqrt{\omega_S^2 + (\omega_C - \omega_S)^2}}{(\omega_S^2 + \omega_C^2)} \sin(\omega_S t + \psi - \theta) \quad \text{and} \quad \tan\theta = \frac{(\omega_C - \omega_S)}{\omega_S}$$

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- ✓ Rewriting the relation

$$-\cos(\omega_S t+\psi)V_g\cos\psi-\sin(\omega_S t+\psi)V_g\sin\psi=-V_g\cos\omega_S t$$

$V_R = V_g \sin(\omega_S t)$

$$F'_{R(peak)}=\frac{V_g\omega_C^2}{(\omega_C^2+\omega_S^2)}$$

$$F''_{R(peak)}=\frac{V_g\omega_C\omega_S}{(\omega_C^2+\omega_S^2)}$$

$$F_R''(t)=\frac{V_g\omega_C\omega_S}{(\omega_S^2+\omega_C^2)}\sin(\omega_S t+\psi)$$

$$F_R'(t)=-\frac{V_g\omega_C^2}{(\omega_S^2+\omega_C^2)}\cos(\omega_S t+\psi)$$

- Substituting
  - ✓ $V_g\cos\psi = 2F''_{R(peak)}$
  - ✓ $V_g\sin\psi\cong 2F'_{R(peak)}-2F''_{R(peak)}$

$$-V_g\cos\omega_S t\cong U_2=-\cos(\omega_S t+\psi)(2F''_{R(peak)})-\sin(\omega_S t+\psi)(2F'_{R(peak)}-2F''_{R(peak)})$$

$$U_2=2F_R''(t)-2\{F''_{R(peak)}\cos(\omega_S t+\psi)\}-2\{F'_{R(peak)}\sin(\omega_S t+\psi)\}$$

$$U_2=-\frac{2V_g\omega_C\sqrt{\omega_S^2+(\omega_C-\omega_S)^2}}{(\omega_S^2+\omega_C^2)}\cos(\omega_S t+\psi-\theta)\quad \text{and}\quad \tan\theta=\frac{(\omega_C-\omega_S)}{\omega_S}$$

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

$V_R = V_g \sin(\omega_S t)$

- Comparing steady state term of $U_1(t)$ and $U_2(t)$

$$U_1 = \frac{2V_g\omega_C\sqrt{\omega_S^{\,2}+(\omega_C-\omega_S)^2}}{(\omega_S^{\,2}+\omega_C^{\,2})}\sin(\omega_S t+\psi-\theta)$$

$$U_2 = -\frac{2V_g\omega_C\sqrt{\omega_S^{\,2}+(\omega_C-\omega_S)^2}}{(\omega_S^{\,2}+\omega_C^{\,2})}\cos(\omega_S t+\psi-\theta)$$

- Inference from the above result
  - ✓ $U_1(t)$ and $U_2(t)$ are $90^0$ displaced
  - ✓ At the zero crossing of $U_1(t)$, $U_2(t)$ will be in peak and vice versa
  - ✓ Unit magnitude of $U_1(t)$ and $U_2(t)$ can be obtained by dividing the term by its own magnitude
  - ✓ $U_1(t)$ is phase shifted from grid voltage by angle $(\psi - \theta)$

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- Inference from the above result (contd.)
  - ✓ Fix the corner frequency of LPF ($\omega_c$) is equal to $\omega_s$ = $2\pi 50$ rad/sec, where grid frequency $f_s$ = 50Hz
  - ✓ Let Grid frequency varies a maximum of $\pm$10% (45Hz to 55Hz)
  - ✓ Give a $\Delta\omega$ variation and substitute $\omega_c = \omega_s$ in the phase angle ($\psi$- $\theta$)

$$\tan(\psi-\theta)=\frac{\tan\psi-\tan\theta}{1+\tan\psi\tan\theta}$$

$$\tan(\psi-\theta)\Big|_{\substack{\omega_S=\omega_S+\Delta\omega \\ \omega_C=\omega_S}} \cong -\frac{\frac{1}{2}\left(\frac{\Delta\omega}{\omega_S}\right)^2}{\left(1+\left(\frac{\Delta\omega}{\omega_S}\right)\right)} \text{ if } 2\left(\frac{\Delta\omega}{\omega_S}\right)^2 \text{ and } \frac{1}{2}\left(\frac{\Delta\omega}{\omega_S}\right)^3 \text{ is negligible compare to } 1$$

$V_R = V_g \sin(\omega_S t)$

$$\tan\theta=\frac{(\omega_C-\omega_S)}{\omega_S}$$

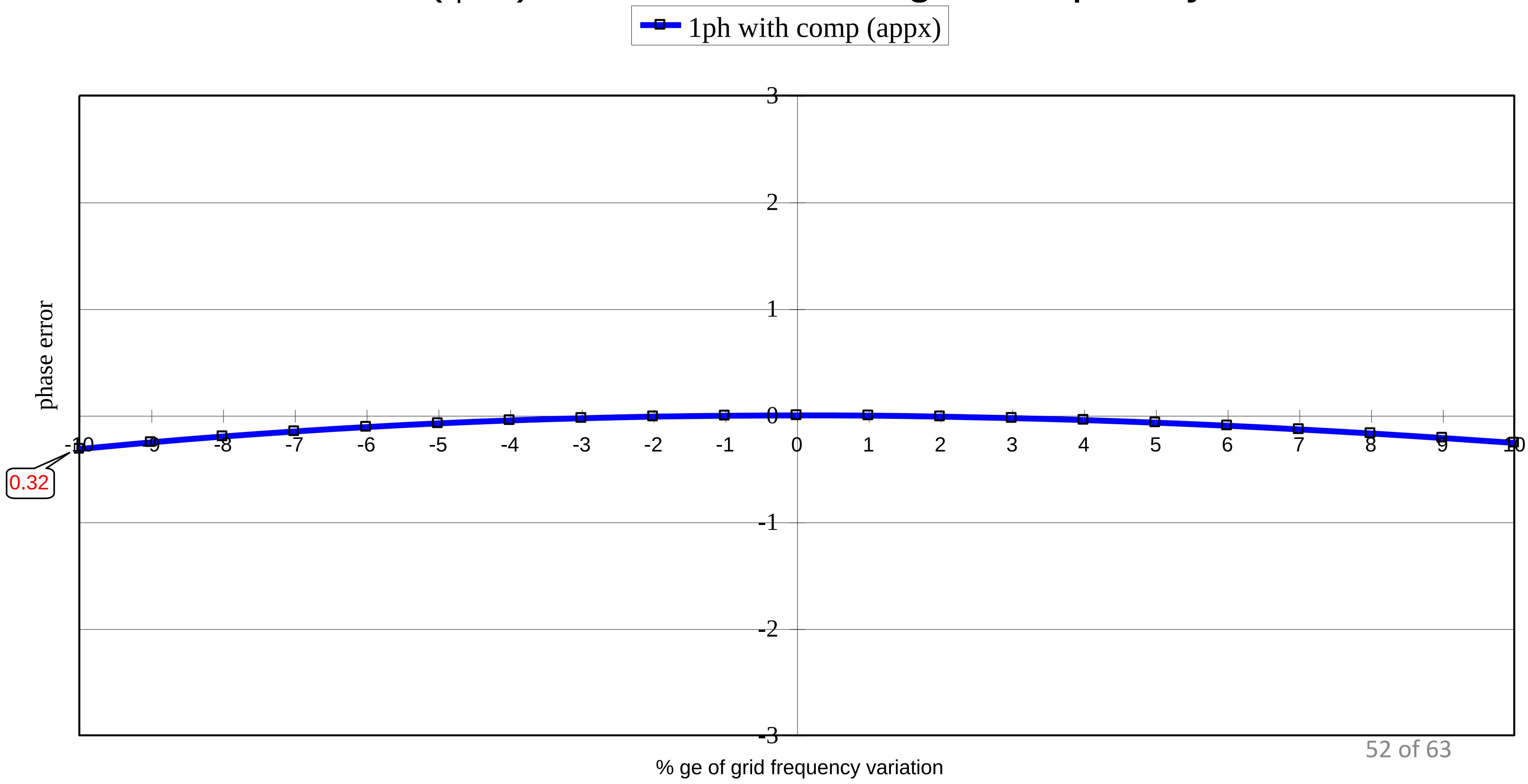
$$\sin\psi=\frac{(\omega_C^2-\omega_S^2)}{(\omega_S^2+\omega_C^2)}$$

$$\cos\psi=\frac{2\omega_C\omega_S}{(\omega_S^2+\omega_C^2)}$$

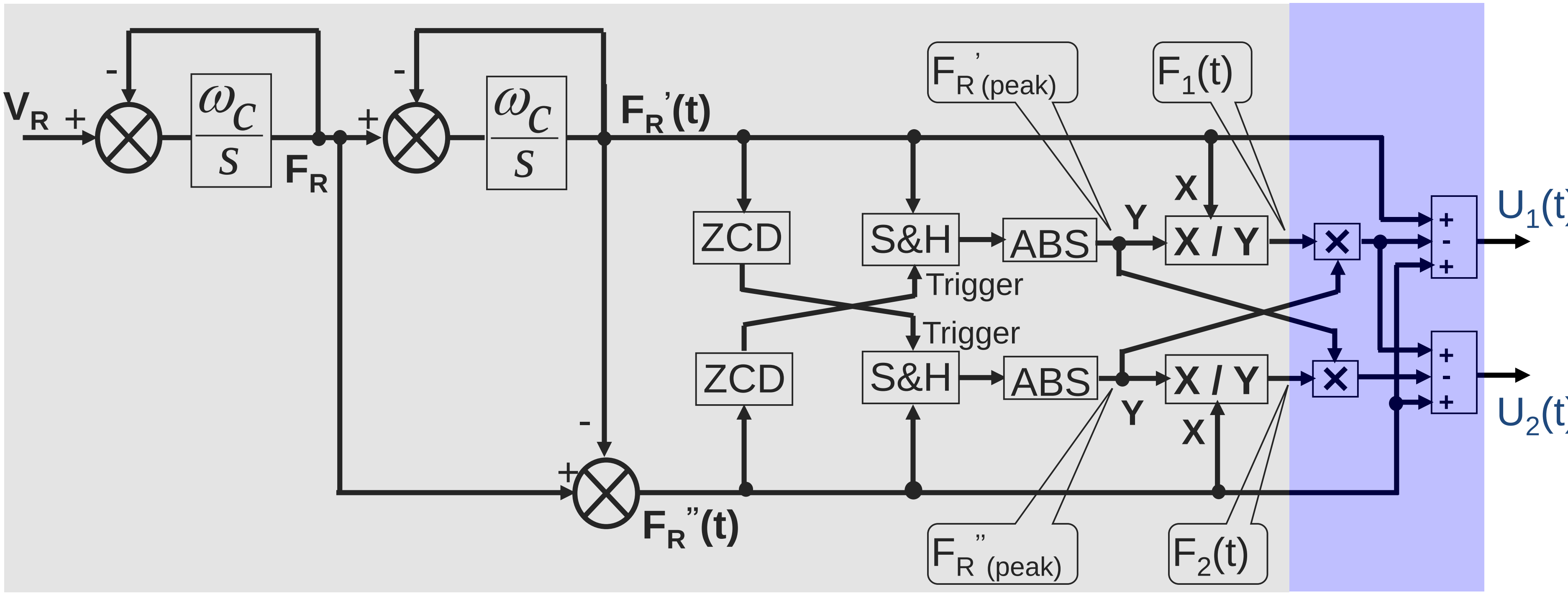
# constructing unit vector (contd.)

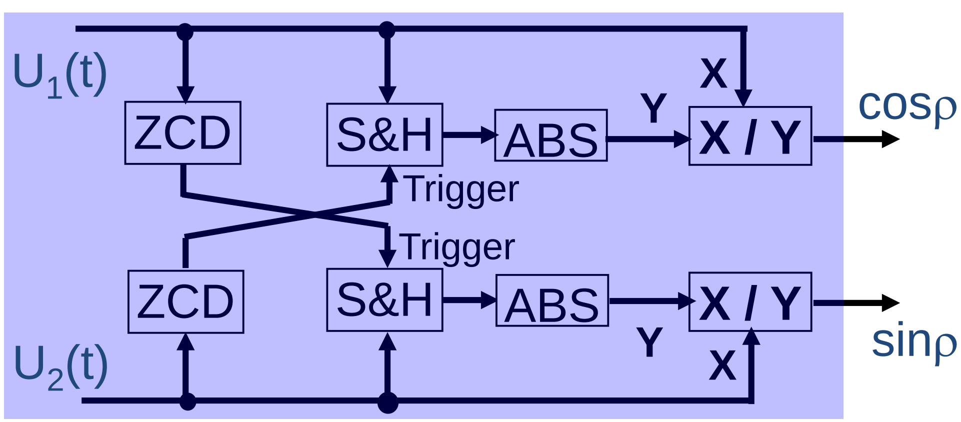
## Mitigating the effect of grid frequency variation (contd.)

- Phase error with variation in grid frequency
  - Phase shift, $(\psi-\theta) = 0.32^0$ for -10% grid frequency variation



# constructing unit vector (contd.)




$$U_1 = 2F_R''(t) + 2F_R'(t) + 2\left[F''_{R(peak)}\cos(\omega_S t + \psi)\right]$$

$$U_2 = 2F_R''(t) - 2\{F''_{R(peak)}\cos(\omega_S t + \psi)\} - 2\{F'_{R(peak)}\sin(\omega_S t + \psi)\}$$

$F_1(t) = -\cos(\omega_S t+\psi)$

$F_2(t) = \sin(\omega_S t+\psi)$

# Overview of the presentation

- Constructing two unit magnitude $90^0$ displaced components
- Mitigating the effect of grid frequency variation
  - ✓ Approximation method
  - ✓ Rigorous method

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

$V_R = V_g \sin(\omega_S t)$

$$\sin\psi = \frac{\left(\omega_C{}^2 - \omega_S{}^2\right)}{\left(\omega_S{}^2 + \omega_C{}^2\right)}$$

$$F''_{R(peak)} = \frac{V_g \omega_C \omega_S}{\left(\omega_C{}^2 + \omega_S{}^2\right)}$$

$$F'_{R(peak)} = \frac{V_g \omega_C{}^2}{\left(\omega_C{}^2 + \omega_S{}^2\right)}$$

- The approximation made in the earlier derivation is:

$$2F'_{R(peak)} - V_g = V_g \sin\psi$$

$$F_R''\Big|_{\substack{\omega_C = \omega_S \\ \omega_S = \omega_S + \Delta\omega}} \cong \frac{V_g}{2} \quad \text{if } \frac{1}{2}\left(\frac{\Delta\omega}{\omega_S}\right)^2 \text{ is negligible compare to 1}$$

- The above method is Approximation method
- Another method of finding out $V_g \sin(\omega_s t)$ is:

$$F'_{R(peak)} + F''_{R(peak)} = \frac{V_g \omega_C}{\left(\omega_S{}^2 + \omega_C{}^2\right)}\left(\omega_C + \omega_S\right) \quad \& \quad F'_{R(peak)} - F''_{R(peak)} = \frac{V_g \omega_C}{\left(\omega_S{}^2 + \omega_C{}^2\right)}\left(\omega_C - \omega_S\right)$$

$$\frac{\left(F'_{R(peak)} + F''_{R(peak)}\right)\left(F'_{R(peak)} - F''_{R(peak)}\right)}{F'_{R(peak)}} = \frac{\dfrac{V_g{}^2 \omega_C{}^2}{\left(\omega_S{}^2 + \omega_C{}^2\right)^2}\left(\omega_C{}^2 - \omega_S{}^2\right)}{\left\{\dfrac{V_g \omega_C{}^2}{\left(\omega_S{}^2 + \omega_C{}^2\right)}\right\}} = V_g \frac{\left(\omega_C{}^2 - \omega_S{}^2\right)}{\left(\omega_S{}^2 + \omega_C{}^2\right)} = V_g \sin\psi$$

# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- Rewriting the relation

$$\sin(\omega_S t+\psi)V_g\cos\psi-\cos(\omega_S t+\psi)V_g\sin\psi=V_g\sin\omega_S t$$

$$-\cos(\omega_S t+\psi)V_g\cos\psi-\sin(\omega_S t+\psi)V_g\sin\psi=-V_g\cos\omega_S t$$

- Substituting
  - ✓ $V_g\cos\psi = 2F''_{R(peak)}$
  - ✓ $$V_g\sin\psi=\frac{\left(F'^{\,2}_{R(peak)}-F''^{\,2}_{R(peak)}\right)}{F'_{R(peak)}}$$

$$U_1=V_g\sin\omega_S t=2F''_R(t)-\left\{\frac{\left(F'^{\,2}_{R(peak)}-F''^{\,2}_{R(peak)}\right)}{F'_{R(peak)}}\right\}\cos(\omega_S t+\psi)$$

$$U_2=-V_g\cos\omega_S t=-2F''_{R(peak)}\cos(\omega_S t+\psi)-\left\{\frac{\left(F'^{\,2}_{R(peak)}-F''^{\,2}_{R(peak)}\right)}{F'_{R(peak)}}\right\}\sin(\omega_S t+\psi)$$

$V_R = V_g \sin(\omega_S t)$

$$F'_{R(peak)}=\frac{V_g\omega_C^2}{\left(\omega_C^2+\omega_S^2\right)}$$

$$F''_{R(peak)}=\frac{V_g\omega_C\omega_S}{\left(\omega_C^2+\omega_S^2\right)}$$

$$U_1=2F_R''(t)+2F_R'(t)+2\left[F''_{R(peak)}\cos(\omega_S t+\psi)\right]$$

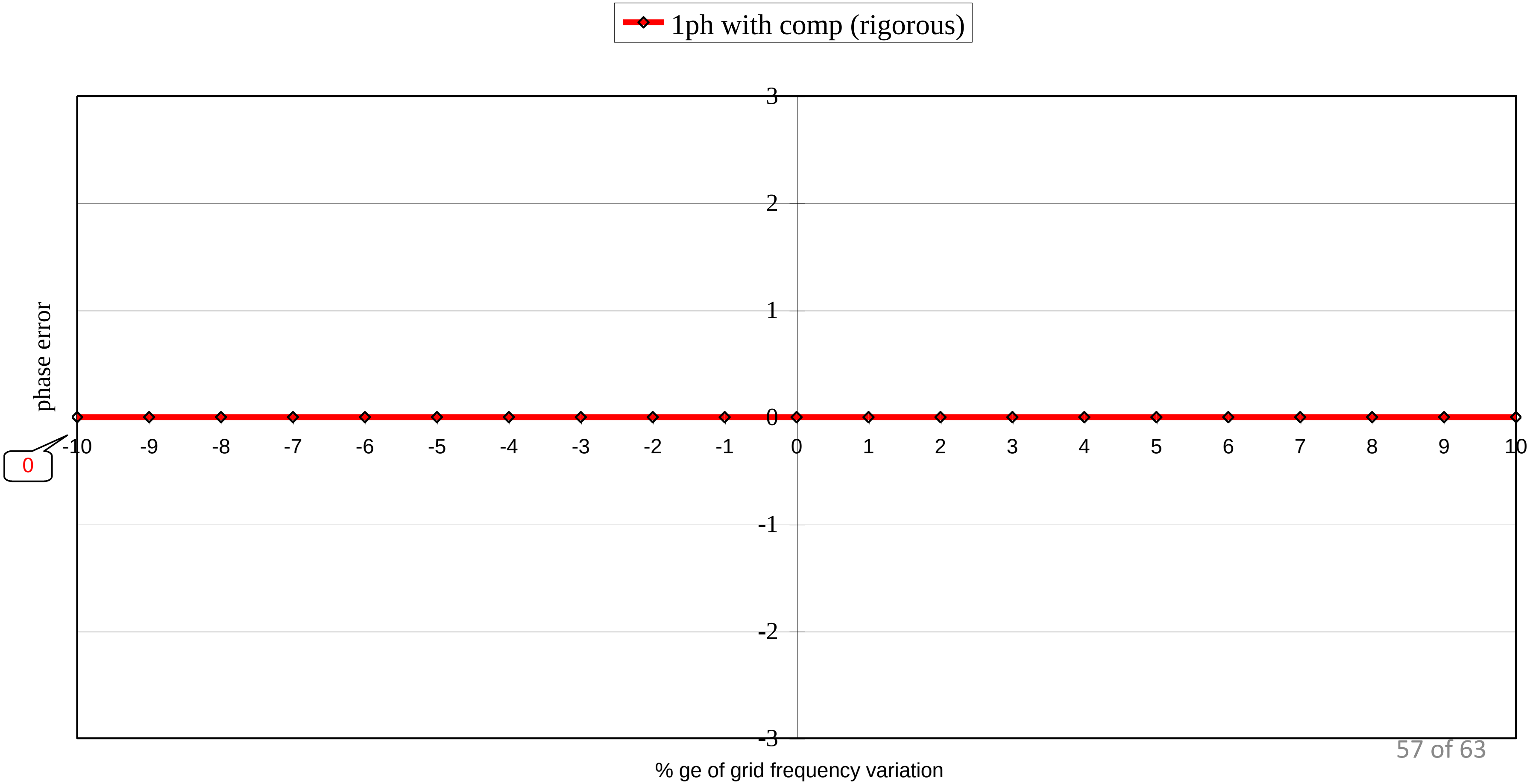
$$U_2=2F_R''(t)-2\{F''_{R(peak)}\cos(\omega_S t+\psi)\}-2\{F'_{R(peak)}\sin(\omega_S t+\psi)\}$$

$$F_R''(t)=\frac{V_g\omega_C\omega_S}{\left(\omega_S^2+\omega_C^2\right)}\sin(\omega_S t+\psi)$$

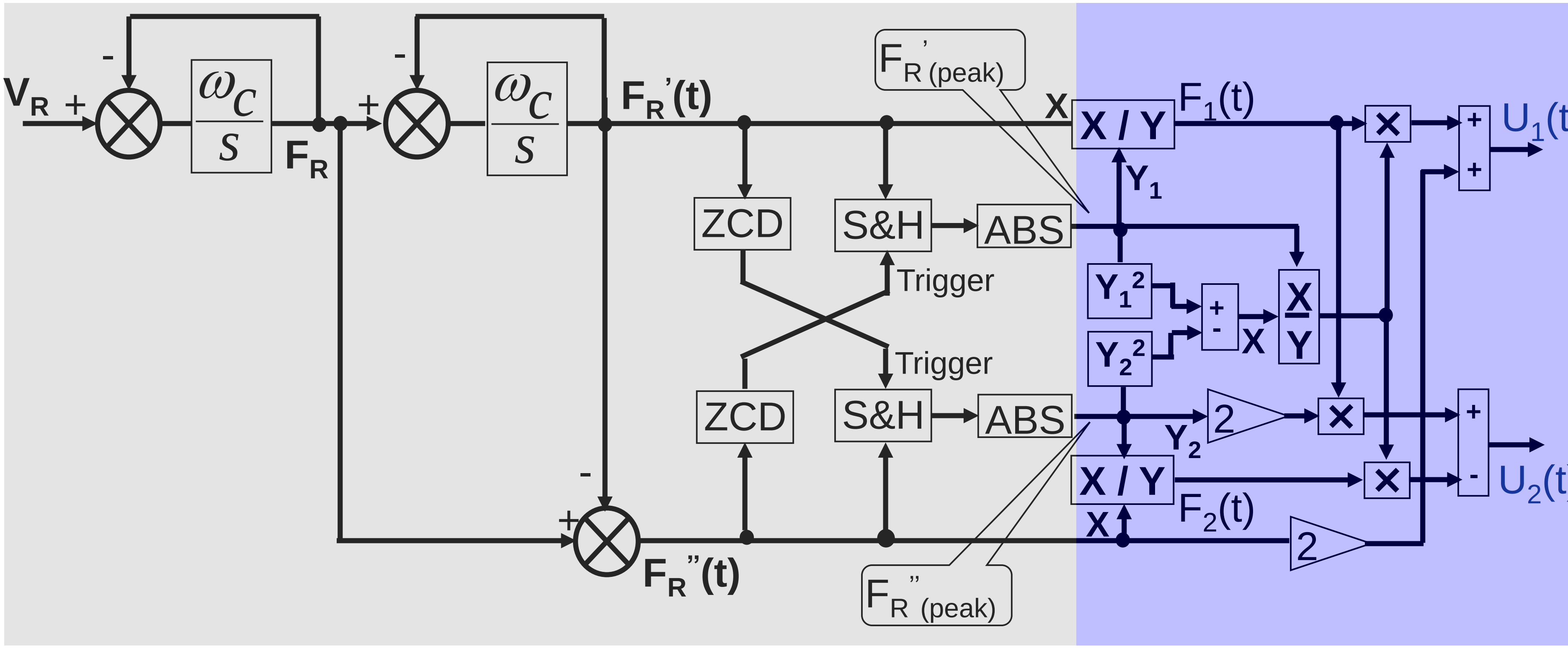
$$F_R'(t)=-\frac{V_g\omega_C^2}{\left(\omega_S^2+\omega_C^2\right)}\cos(\omega_S t+\psi)$$

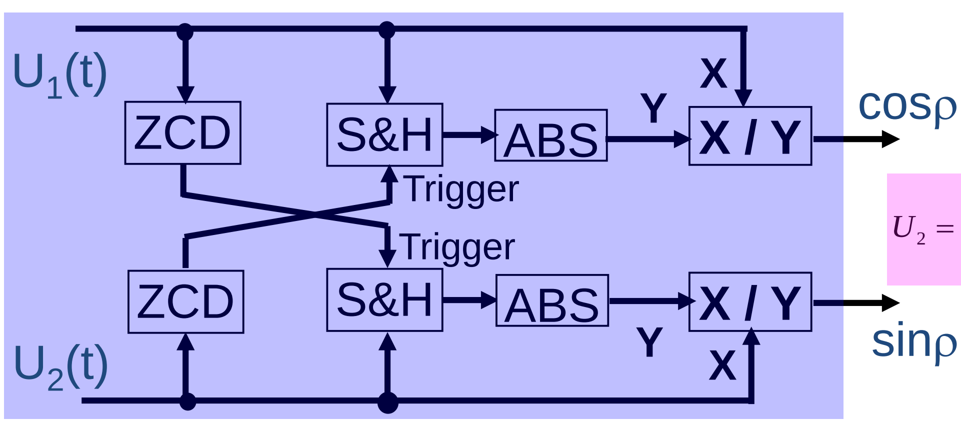
## Mitigating the effect of grid frequency variation (contd.)

- Phase error with variation in grid frequency
  - Phase shift, $(\psi-\theta) = 0^0$ for any grid frequency



# constructing unit vector (contd.)




$$U_1 = V_g \sin \omega_S t = 2F_R''(t) - \left\{ \frac{\left( {F'_{R(peak)}}^2 - {F''_{R(peak)}}^2 \right)}{F'_{R(peak)}} \right\} \cos(\omega_S t + \psi)$$

$$U_2 = -V_g \cos \omega_S t = -2F''_{R(peak)} \cos(\omega_S t + \psi) - \left\{ \frac{\left( {F'_{R(peak)}}^2 - {F''_{R(peak)}}^2 \right)}{F'_{R(peak)}} \right\} \sin(\omega_S t + \psi)$$

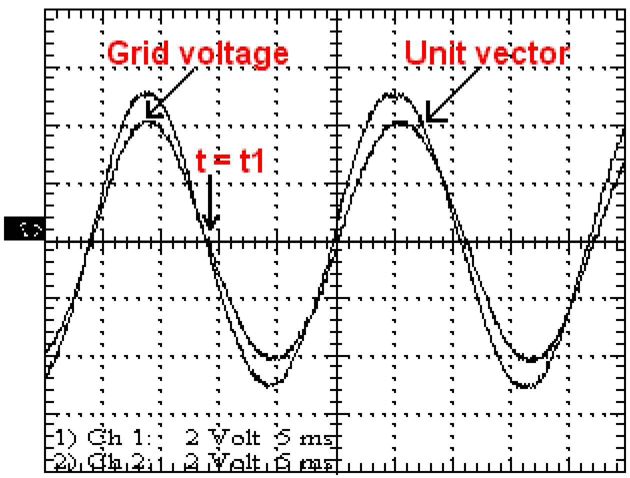
$F_1(t) = -\cos(\omega_S t + \psi)$

$F_2(t) = \sin(\omega_S t + \psi)$

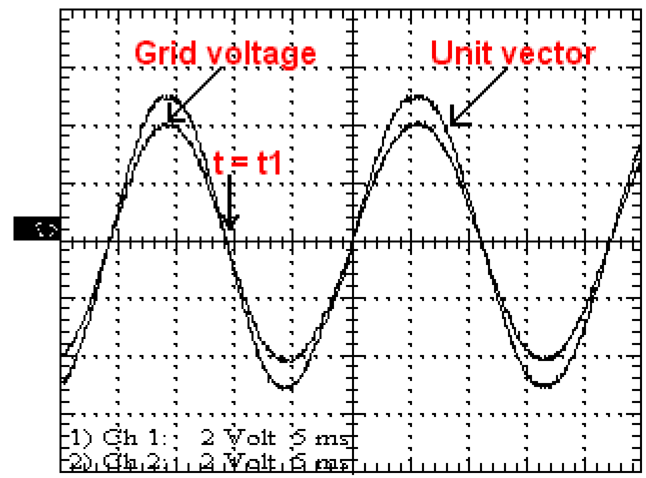
# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- Test results
  - ✓ Grid frequency (simulated using function generator) is varied at t=t1 from 50Hz to 45Hz




Grid voltage and unit vector without compensation for grid frequency variation

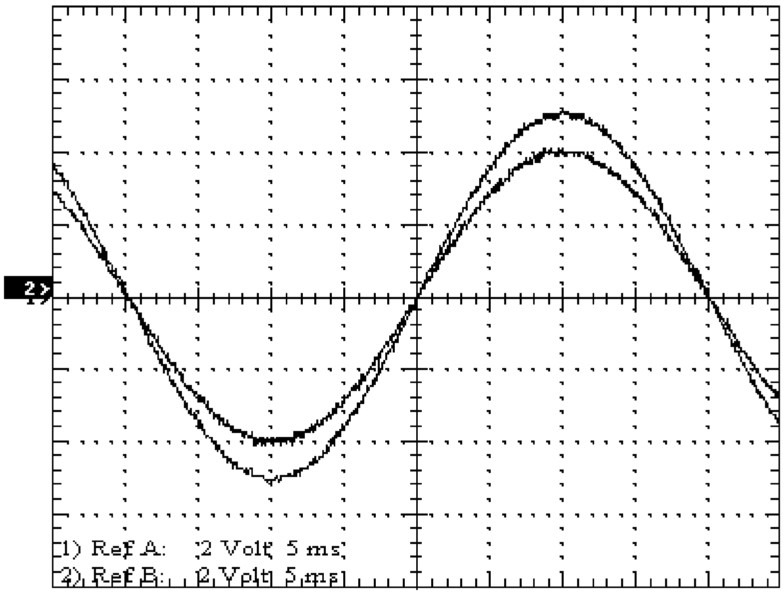



Grid voltage and unit vector with compensation for grid frequency variation

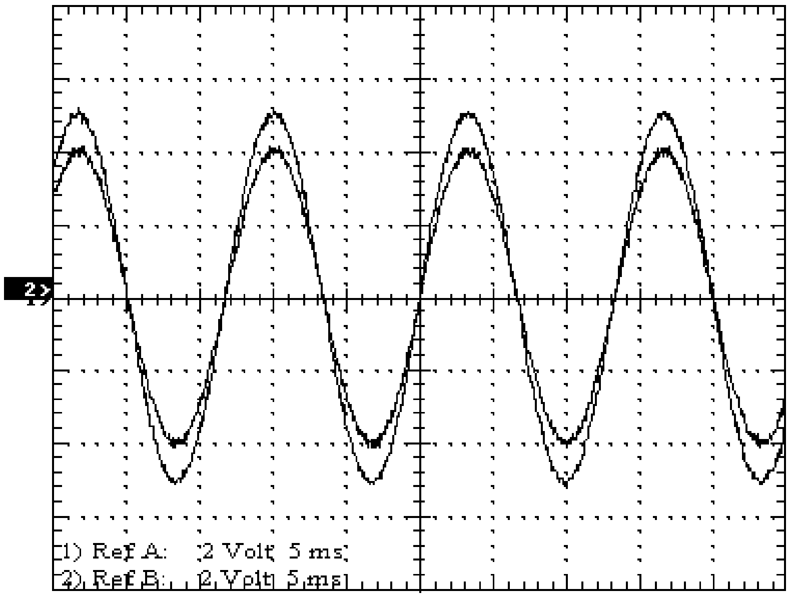
# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- Test results
  - Steady state waveform at 25Hz and 75Hz with approximation method




Frequency = 25Hz

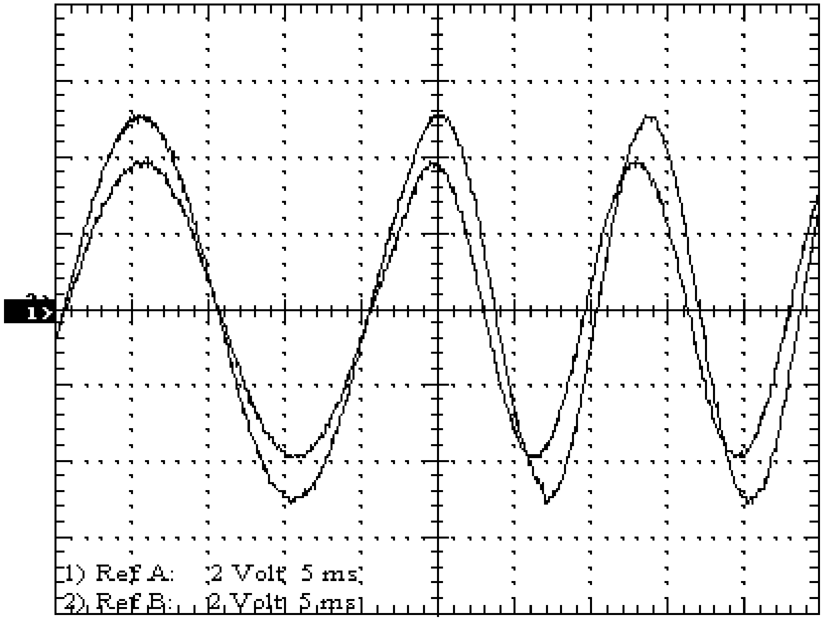



Frequency = 75Hz

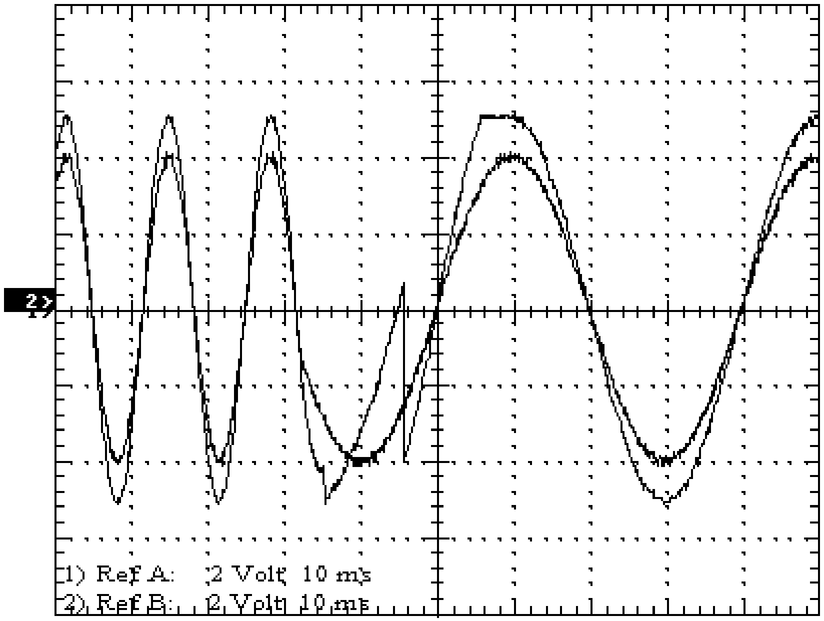
# constructing unit vector (contd.)

## Mitigating the effect of grid frequency variation (contd.)

- Test results
  - Step change of frequency from 25Hz and 75Hz as well as from 75Hz to 25Hz




Grid voltage and unit vector without compensation for grid frequency variation




Grid voltage and unit vector with compensation for grid frequency variation

# Conclusions

- Single grid voltage is considered for the construction of unit vector
  - Two unity magnitude fundamental sinusoidal quantities, which are displaced by $90^0$ from each other
  - One of the unit vector is in phase with grid voltage irrespective of the grid frequency variation

*Thank you*